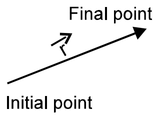
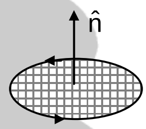
ELEMENTS OF VECTORS

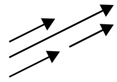
1. **Scalar :** A physical quantity having only magnitude but not associated with any direction is called a scalar.
   eg: time, mass, distance, speed, work, energy, power, pressure, temperature, electric current, gravitational potential, pole strength, magnetic flux, entropy, electric capacity, velocity of light, large angular displacement, electric charge, etc.
2. Scalars are added and subtracted by algebraic method.
3. **Vector :** A physical quantity having magnitude as well as associated direction and which obeys vector laws is called a vector.
   eg: displacement, velocity, acceleration, force, momentum, impulse, moment of force, small angular displacement, angular velocity, angular acceleration, magnetic moment, dipole moment, current density, intensity of electric field or magnetic field, shearing stress, weight, centrifugal force, infinitesimally small area, etc.
4. Vectors are completely described by a number with a unit followed by a statement of direction.

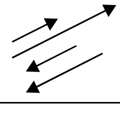


5. Angle can be considered as vector if it is small. Large angles can not be treated as vectors as they do not obey laws of vector addition.
6. Surface area can be treated both as a scalar and a vector. A is magnitude of surface area which is a scalar. This area is enclosed by a closed curve as shown if $\hat{n}$ is a unit vector normal to the surface, we can write $A\,\hat{n}$ as a vector.



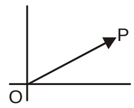
   $\Rightarrow$ Surface area is a vector.
   [If the four fingers of right hand curl along the direction of arrow of enclosing curve, thumb indicates direction of area vector]
7. Tensor is a physical quantity which will have different values along different directions.
   e.g. Moment of inertia, stress.
8. A vector is represented by a directed line segment. The length of the line segment is proportional to the magnitude of the vector.
9. The magnitude or modulus of a vector ($|\vec{r}|$ or r) is a scalar.
10. Electric current, velocity of light has both magnitude and direction but they do not obey the laws of vector addition. Hence they are scalars.
11. **Equal vectors :** Two vectors are said to be equal if they have the same magnitude and direction irrespective of their initial points.
12. **Negative vectors :** $\vec{A}$ and $-\vec{A}$ are vectors having the same magnitude and opposite direction. $-\vec{A}$ is called the negative of $\vec{A}$.
13. **Proper vector :** A vector whose magnitude is not zero is known as proper vector.
14. **Null Vector (Zero Vector):** It is a vector whose magnitude is zero and direction is unspecified.
    Examples:
    a) Displacement after one complete revolution.
    b) Velocity of vertically projected body at the highest point
15. **Parallel vectors :** Vectors in the same direction are called parallel vectors.
16. **Antiparallel vectors :** Vectors in opposite direction are called antiparallel vectors.
17. **Like vectors or co-directional vectors :** The vectors directed in the same direction, irrespective of their magnitudes are called co-directional vectors or like vectors.





18. **Collinear vectors :** Two or more vectors parallel or antiparallel to each other are called collinear vectors.
19. **Coplanar vectors :** Vectors lying on the same plane are called coplanar vectors and the plane in which they lie is called the plane of the vectors.
20. **Unit vector :** It is a vector whose magnitude is unity. A unit vector parallel to a given vector.
21. If $\vec{A}$ is a vector, the unit vector in the direction of $\vec{A}$ is written as $\vec{a}$ or $\hat{A} = \frac{\text{vector } \vec{A}}{\text{modulus of } \vec{A}} = \frac{\vec{A}}{|\vec{A}|}$.

    $\hat{i}, \hat{j}$ and $\hat{k}$ are units vectors along x, y and z axis.
22. **Position vector :** The vector which is used to specify the position of a point 'P' with respect to some fixed point 'O' is represented by $\overrightarrow{OP}$ and is known as the position vector of 'P' with respect to 'O'.



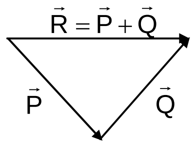
23. **Real Vector or Polar Vector:** If the direction of a vector is independent of the coordinate system., then it is called a polar vector. Example: linear velocity, linear momentum, force, etc.,
24. **Pseudo or axial vectors :** Axial vectors or pseudo vectors are those whose direction is fixed by convention and reverses in a mirror reflection. Cross product of two vectors gives an axial vector. eg : Torque, angular velocity, etc.
25. A vector remains unchanged when it is moved parallel to itself.
26. If m is a scalar and $\vec{A}$ a vector, then m$\vec{A}$ is a vector. Its magnitude is m times that of magnitude of $\vec{A}$. Its direction is the same as that of $\vec{A}$, if m is positive and opposite if m is negative,

    $\vec{P} = m\vec{V}$; $\vec{F} = m\vec{a}$;

    $\vec{F} = \vec{E}q$; $\vec{F} = m\vec{B}$. If m is zero, m$\vec{A}$ is a null vector.
27. Vector multiplication obeys commutative law when multiplied by a scalar. $s\vec{A} = \vec{A}s$ where s is scalar.
28. Vector multiplication obeys associative law when multiplier by a scalar i.e. $m(n\vec{A}) = mn\vec{A}$ (m, n are scalars)
29. Vector multiplication obeys distributive law when multiplied by a scalar. $s(\vec{A} + \vec{B}) = s\vec{A} + s\vec{B}$.

ADDITION OF VECTORS:

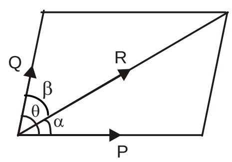
30. Addition of vectors is also called resultant of vectors.
31. **Resultant** is a single vector that gives the total effect of number of vectors.
    Resultant can be found by using
    a) Triangle law of vectors
    b) Parallelogram law of vectors
    c) Polygon law of vectors
32. Two vectors can be added either by triangle law or parallelogram law of vectors.
33. **Triangle law :** If two vectors are represented in magnitude and direction by the two sides of a triangle taken in order, then the third side taken in the reverse order represents their sum or resultant in magnitude as well as in direction.



34. **Parallelogram law :**

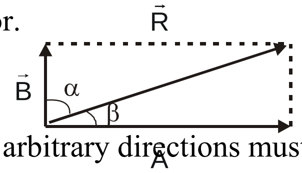
    If two vectors $\vec{P}$ and $\vec{Q}$ are represented by the two sides of a parallelogram drawn from a point, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.

    $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$



$\text{Tan}\,\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$; $\tan\beta = \dfrac{P\sin\theta}{Q + P\cos\theta}$

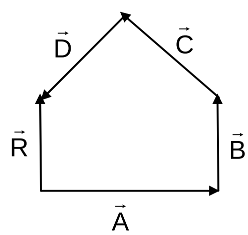
35. The resultant of two vectors is the vectorial addition of two vectors.
36. The resultant of any two vectors makes lesser angle with the greater vector.



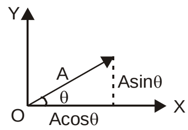
37. If $|\vec{A}| > |\vec{B}|$ $\quad \alpha < \beta$
38. The magnitude of the resultant of two vectors of magnitudes a and b with arbitrary directions must be in the range (a – b) to (a + b).
39. $\vec{a}$ and $\vec{b}$ are two vectors which when added give a vector $\vec{c}$ (i.e.,) $\vec{a} + \vec{b} = \vec{c}$ and if
    i) $|\vec{a}| + |\vec{b}| = |\vec{c}|$ then $\vec{a}$ and $\vec{b}$ are parallel vectors ($\theta = 0°$)
    ii) $|\vec{a}|^2 + |\vec{b}|^2 = |\vec{c}|^2$ then $\vec{a}$ and $\vec{b}$ are perpendicular vectors ($\theta = 90°$)
    iii) $|\vec{a}| - |\vec{b}| = |\vec{c}|$ then $\vec{a}$ and $\vec{b}$ are antiparallel vectors ($\theta = 180°$)
    iv) $|\vec{a}| = |\vec{b}| = |\vec{c}|$ then $\vec{a}$ and $\vec{b}$ are inclined to each other at 120°
    v) If $|\vec{A}| = |\vec{B}|$ and $|\vec{A} + \vec{B}| = |\vec{A}|$, then $\theta = 120°$.
    vi) If $|\vec{A}| = |\vec{B}|$ and $|\vec{A} - \vec{B}| = |\vec{A}|$, then $\theta = 60°$.
    vii) If $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, then $\theta = 90°$.
40. If two vectors each of magnitude F act at a point, the magnitude of their resultant (R) depends on the angle θ between them.
    $R = 2F\cos(\theta/2)$.

Angle between forces (θ)	**Magnitude of resultant**
0°	2F
60°	$\sqrt{3}$ F
90°	$\sqrt{2}$ F
120°	F
180°	0

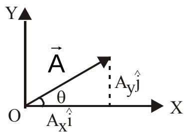
41. Minimum number of equal vectors to give a zero resultant is 2.
42. The minimum number of unequal vectors to give a zero resultant is 3.
43. There are three laws of addition of vectors.
    a) Commutative law: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$
    b) Associative law: $\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$
    c) Distributive law: $m(\vec{A} + \vec{B}) = m\vec{A} + m\vec{B}$ where m is a scalar.
44. If the number of vectors is more than two, polygon law of vectors is used.
45. **Polygon law :** If a number of vectors are represented by the sides of a polygon taken in the same order, the resultant is represented by the closing side of the polygon taken in the reverse order.
    $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$



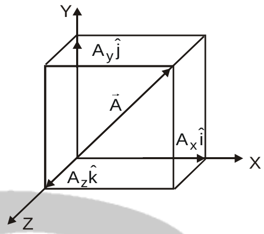
46. **Resolution of a vector in two dimensions :** If $\vec{A}$ is a vector making an angle θ with x–axis, then X–component = $A\cos\theta$, Y–component = $A\sin\theta$.



47. If $\hat{i}$ and $\hat{j}$ are unit vectors along X and Y axes, any vector lying in XOY plane can be represented as $\vec{A} = A_x\hat{i} + A_y\hat{j}$;



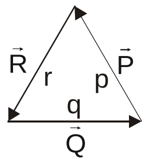
48. $|\vec{A}| = A = \sqrt{A_x^2 + A_y^2}$; $\tan\theta = \dfrac{A_y}{A_x}$
49. The component of a vector can have a magnitude greater than that of the vector itself.
50. The rectangular component cannot have magnitude greater than that of the vector itself.
51. If a number of vectors $\vec{A}, \vec{B}, \vec{C}, \vec{D}$,..... acting at a point are resolved along X–direction as A_x, B_x, C_x, D_x, … along Y–direction as A_y, B_y, C_y, D_y….. and if $\vec{R}$ is the resultant of all the vectors, then the components of $\vec{R}$ along X–direction and
Y–direction are given by $R_x = A_x + B_x + C_x + D_x + \ldots$ and $R_y = A_y + B_y + C_y + D_y + \ldots$ respectively, and
$R = \sqrt{R_x^2 + R_y^2}$; $\tan\theta = \dfrac{R_y}{R_x}$ where θ is the angle made by the resultant with X–direction.



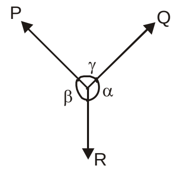
52. If $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along X, Y and Z–axes, any vector in 3 dimensional space can be expressed as
$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$; $|\vec{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$ Here A_x, A_y, A_z are the components of $\vec{A}$ and $\vec{B}$ are scalars. $\vec{A}$ is body diagonal of the cube.
53. If α, β and γ are the angles made by $\vec{A}$ with X–axis, Y–axis and Z–axis respectively, then
$\cos\alpha = \dfrac{A_x}{A}$; $\cos\beta = \dfrac{A_y}{A}$; $\cos\gamma = \dfrac{A_z}{A}$ and
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

54. If $\cos\alpha = l$, $\cos\beta = m$ and $\cos\gamma = n$, then l, m, n are called direction cosines of the vector. $l^2 + m^2 + n^2 = 1$.
55. If vectors $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ are parallel, then $\dfrac{A_x}{B_x} = \dfrac{A_y}{B_y} = \dfrac{A_z}{B_z}$ and $\vec{A} = K\vec{B}$
where K is a scalar.
56. The vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the coordinate axes at an angle of 54.74°.
57. The position vector of a point P(x,y,z) is given by
$\overrightarrow{OP} = x\hat{i} + y\hat{j} + z\hat{k}$ and $|\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2}$
58. The vector having initial point $P(x_1, y_1, z_1)$ and final point $Q(x_2, y_2, z_2)$ is given by
$\overrightarrow{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$.
59. **Equilibrium** is the state of a body in which there is no acceleration i.e., net force acting on a body is zero.
60. The forces whose lines of action pass through a common point (called the point of concurrence) are called **concurrent** forces.

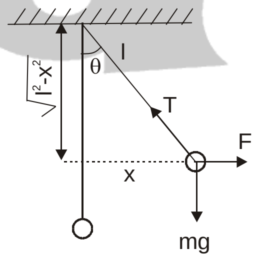
61. **Resultant force** is the single force which produces the same effect as a given system of forces acting simultaneously.
62. A force which when acting along with a given system of forces produces equilibrium is called the **equilibrant**.
63. **Resultant and equilibrant** have equal magnitude and opposite direction. They act along the same line and they are themselves in equilibrium.
64. **Triangle law of forces :** If a body is in equilibrium under the action of three coplanar forces, then these forces can be represented in magnitude as well as direction by the three sides of a triangle taken in order. $\frac{p}{|\vec{P}|} = \frac{q}{|\vec{Q}|} = \frac{r}{|\vec{R}|}$ where p, q, r are sides of a triangle. $\vec{P}, \vec{Q}, \vec{R}$ are coplanar vectors.



65. **Lami's theorem :** When three coplanar forces $\vec{P}, \vec{Q}$ and $\vec{R}$ keep a body in equilibrium, then $\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$.



66. When a number of forces acting on a body keep it in equilibrium, then the algebraic sum of the components along the X–direction is equal to zero and the algebraic sum of the components along the Y–direction is also equal to zero. i.e., $\Sigma F_x = 0$ and $\Sigma F_y = 0$.
67. If $\vec{A}_1 + \vec{A}_2 + \vec{A}_3 + ... + \vec{A}_n = 0$ and $A_1 = A_2 = A_3 = \dots A_n$, then the adjacent vectors are inclined to each other at an angle $\frac{2\pi}{N}$ or $\frac{360°}{N}$.
68. N forces each of magnitude F are acting on a point and angle between any two adjacent forces is θ, then resultant force $F_{resultant} = \frac{F\sin\left(\frac{N\theta}{2}\right)}{\sin(\theta/2)}$.
69. **BODY PULLED HORIZONTALLY :**
    i) A body is suspended by a string from a rigid support. It is pulled aside so that it makes an angle θ with the vertical by applying a horizontal force F. When the body is in equilibrium,
    ii) Horizontal force,
    F= mgTanθ
    iii) Tension in the string,
    $T = \frac{mg}{\cos\theta}$
    iv) $T = \sqrt{(mg)^2 + F^2}$
    v) $\frac{T}{l} = \frac{mg}{\sqrt{l^2 - x^2}} = \frac{F}{x}$

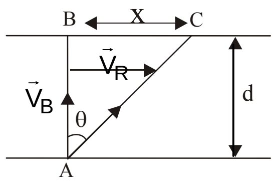


70. If a body simultaneously possesses two velocities $\vec{u}$ and $\vec{v}$, the resultant velocity is given by the following formulae.
    a) If $\vec{u}$ and $\vec{v}$ are in the same direction, the resultant velocity will be $|\vec{u}| + |\vec{v}|$ in the direction of $\vec{u}$ or $\vec{v}$.
    b) If $\vec{u}$ and $\vec{v}$ are in opposite direction, the magnitude of the resultant velocity will be $|\vec{u}| \sim |\vec{v}|$ and acts in the direction of the greater velocity.
    c) If $\vec{u}$ and $\vec{v}$ are mutually perpendicular, the resultant velocity will be $\sqrt{u^2 + v^2}$ making an angle $\tan^{-1}(v/u)$ with the direction of $\vec{u}$.

71. If $\vec{v}_1$ is the velocity of the flow of water in a river and $\vec{v}_2$ is the velocity of a boat (relative to still water), then the velocity of the boat w.r.t the ground is $\vec{v}_1 + \vec{v}_2$.
 i. If the boat is going down stream, the velocity of the boat relative to the ground, $|\vec{v}| = |\vec{v}_1| + |\vec{v}_2|$.
 ii. If the boat is going upstream, the velocity of the boat relative to the ground, $|\vec{v}| = |\vec{v}_1| - |\vec{v}_2|$.
 iii. If the boat is moving at right angles to the stream, $|\vec{v}| = \sqrt{v_1^2 + v_2^2}$ making an angle of $\tan^{-1}(v_1/v_2)$ with the original direction of the motion.

72. **MOTION OF A BOAT CROSSING THE RIVER IN SHORTEST TIME :**

If $\vec{V}_B$ and $\vec{V}_R$ are the velocities of a boat and river flow respectively then to cross the river in shortest time, the boat is to be rowed across the river i.e., along normal to the banks of the river.



i) The direction of the resultant is $\theta = \tan^{-1}\left(\frac{V_R}{V_B}\right)$ with the normal or tan $\theta = \frac{V_R}{V_B} = \frac{x}{d}$

ii) Magnitude of the resultant velocity $v = \sqrt{v_B^2 + v_R^2}$

iii) Time taken to cross the river, $t = \frac{d}{v_B}$ where

d=width of the river or $t = \frac{\sqrt{d^2 + x^2}}{\sqrt{V_B^2 + V_R^2}} = \frac{x}{V_R}$

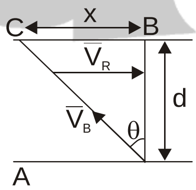
iv) This time is independent of velocity of the river flow.

v) The distance travelled down stream = BC

$x = \frac{d}{V_B} \times V_R$

73. **MOTION OF A BOAT CROSSING THE RIVER IN SHORTEST DISTANCE :**

i) The boat is to be rowed upstream making some angle θ with normal to the bank of the river which is given by $\theta = \sin^{-1}\left(\frac{V_R}{V_B}\right)$ or $\sin\theta = \frac{V_R}{V_B}$



ii) The angle made by boat with the bank or river current is (90° + θ)

iii) Resultant velocity has a magnitude of

$V = \sqrt{V_B^2 - V_R^2}$

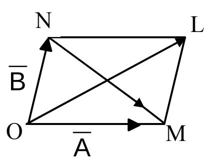
iv) The time taken to cross the river is $t = \frac{d}{\sqrt{V_B^2 - V_R^2}}$

Subtraction of two vectors :

74. If $\vec{P}$ and $\vec{Q}$ are two vectors, then $\vec{P} - \vec{Q}$ is defined as $\vec{P} + (-\vec{Q})$ where $-\vec{Q}$ is the negative vector of $\vec{Q}$.

If $\vec{R} = \vec{P} - \vec{Q}$, then $R = \sqrt{P^2 + Q^2 - 2PQ\cos\theta}$

In the parallelogram OMLN, the diagonal OL represents $\vec{A} + \vec{B}$ and the diagonal NM represents $\vec{A} - \vec{B}$



a) subtraction of vectors does not obey commutative law $\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$

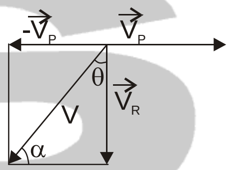
b) subtraction of vectors does not obey Associative law $\vec{A} - (\vec{B} - \vec{C}) \neq (\vec{A} - \vec{B}) - \vec{C}$

c) subtraction of vectors obeys distributive law $m(\vec{A}-\vec{B}) = m\vec{A} - m\vec{B}$

75. If two vectors each of magnitude 'F' act at a point, the magnitude of their difference depends on the angle 'θ' between then

    Magnitude of difference of vectors = $2F\sin\left(\frac{\theta}{2}\right)$

76. **Relative velocity :** When the distance between two bodies is altering either in magnitude or direction or both, then each is said to have a relative velocity with respect to the other.
    Relative velocity is vector difference of velocities.
    a. The relative velocity of body 'A' w.r.t. 'B' is given by $\vec{V}_R = \vec{V}_A - \vec{V}_B$
    b. The relative velocity of body 'B' w.r.t. 'A' is given by $\vec{V}_R = \vec{V}_B - \vec{V}_A$
    c. $\vec{V}_A - \vec{V}_B$ and $\vec{V}_B - \vec{V}_A$ are equal in magnitude but opposite in direction
    d. $\left|\vec{V}_R\right| = \left|\vec{V}_A - \vec{V}_B\right| = \sqrt{V_A^2 + V_B^2 - 2.V_A V_B.\cos\theta}$
    e. For two bodies moving in the same direction, relative velocity is equal to the difference of velocities. ($\theta = 0°.\cos 0 = 1$)
       $\left|\vec{V}_R\right| = V_A - V_B$
    f. For two bodies moving in opposite direction, relative velocity is equal to the sum of their velocities. ($\theta = 180°;\cos 180 = -1$)
       $\therefore \left|\vec{V}_R\right| = V_A + V_B$
    g. If they move at right angle to each other, then the relative velocity $= \sqrt{v_1^2 + v_2^2}$.



77. Rain is falling vertically downwards with a velocity $\vec{V}_R$ and a person is travelling with a velocity $\vec{V}_P$. Then the relative velocity of rain with respect to the person is $\vec{V} = \vec{V}_R - \vec{V}_P$.

    Relative velocity = $|\vec{V}| = \sqrt{V_R{}^2 + V_P{}^2}$.

78. The direction of relative velocity (or) the angle with the vertical at which an umbrella is to be held is given by Tanθ = $\left|\frac{\vec{V}_P}{\vec{V}_R}\right|$.

79. If the product of two vectors is another vector, such a product is called **vector product** or **cross product**.

80. If the product of two vectors is a scalar, then such a product is called **scalar product** or **dot product**.

DOT PRODUCT :

81. **Dot product** is the product of one vector and the component of another vector in its direction. Eg : Magnetic flux, instantaneous power, work done, potential energy.

82. **Dot product** of two vectors $\vec{A}$ and $\vec{B}$ = $\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta = AB\cos\theta$
    a) Scalar product is commutative i.e., $\vec{a}.\vec{b} = \vec{b}.\vec{a}$
    b) Scalar product is distributive i.e., $\vec{a}.(\vec{b}+\vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$

83. If $\vec{A}$ and $\vec{B}$ are parallel vectors, then $\vec{A}.\vec{B}$ = AB.

84. If $\vec{A}$ and $\vec{B}$ are perpendicular to each other, then $\vec{A}.\vec{B}$ =0.
85. If $\vec{A}$ and $\vec{B}$ are antiparallel vectors, then $\vec{A}.\vec{B} = -AB$.
86. Dot product of two vectors may be positive or negative. If θ<90°, it is positive and 90°<θ<270° it is negative.
87. In the case of unit vectors,
$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$ and $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$
88. If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$, then $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$, $\vec{A}.\vec{A} = A_x^2 + A_y^2 + A_z^2$.

APPLICATIONS OF DOT PRODUCT :

89. W = $\vec{F}.\vec{S}$ (dot product of force and displacement is work)
90. P = $\vec{F}.\vec{V}$ (dot product of force and velocity is power)
91. E_p = $m\vec{g}.\vec{h}$ (dot product of gravitational force and vertical displacement is P.E)
92. Magnetic flux, ϕ = $\vec{A}.\vec{B}$ (dot product of area vector and magnetic flux density vector)
93. Angle between the two vectors $\vec{a}$ and $\vec{b}$ is given by Cosθ = $\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$
94. The magnitude of component of vector $\vec{B}$ along vector $\vec{A} = \frac{\vec{A}.\vec{B}}{|\vec{A}|}$.
95. The magnitude of component of vector $\vec{A}$ along vector $\vec{B} = \frac{\vec{A}.\vec{B}}{|\vec{B}|}$.
96. Component of vector $\vec{A}$ along vector $\vec{B}$ = $\frac{\vec{A}.\vec{B}}{|\vec{B}|}.\hat{B}$.

CROSS PRODUCT :

97. The vector or cross product of two vectors $\vec{A}$ and $\vec{B}$ is a vector $\vec{C}$ whose magnitude is AB sin θ where θ is the angle between the vectors $\vec{A}$ and $\vec{B}$ and the direction of $\vec{C}$ is perpendicular to both $\vec{A}$ and $\vec{B}$ such that $\vec{A}, \vec{B}$ and $\vec{C}$ form a right hand triple.

Eg: angular momentum ($\vec{L} = \vec{r} \times \vec{\omega}$), torque ($\vec{\tau} = \vec{r} \times \vec{F}$), angular velocity ($\vec{V} = \vec{\omega} \times \vec{r}$) etc.

In the case of unit vectors
$\hat{i} \times \hat{j} = \hat{k}$; $\hat{j} \times \hat{k} = \hat{i}$; $\hat{k} \times \hat{i} = \hat{j}$;
$\hat{j} \times \hat{i} = -\hat{k}; \hat{k} \times \hat{j} = -\hat{i}; \hat{i} \times \hat{k} = -\hat{j}$ and
$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$.

98. If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$, then $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} =$

$(A_yB_z - A_zB_y)\hat{i} - (A_xB_z - A_zB_x)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$

99. If $\vec{A} \times \vec{B} = 0$ and $\vec{A}$ and $\vec{B}$ are not null vectors, then they are parallel to each other.
$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$;
$m(\vec{A} \times \vec{B}) = (m\vec{A}) \times \vec{B} = \vec{A} \times (m\vec{B})$
a) $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$
(commutative law is not obeyed)
b) $\vec{A} \times (\vec{B} \times \vec{C}) \neq (\vec{A} \times \vec{B}) \times \vec{C}$

(Associate law is not obeyed)

c) $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

(Distributive law is obeyed)

APPLICATIONS OF CROSS PRODUCT :

100.Torque is the cross product of radius vector and force vector, $\vec{\tau} = \vec{r} \times \vec{F}$

101.Angular momentum is the cross product of radius vector and linear momentum, $\vec{L} = \vec{r} \times \vec{p}$

102.Linear velocity in circular motion may be defined as the cross product of angular velocity and radius vector. $\vec{V} = \vec{\omega} \times \vec{r}$

103.The area of the triangle formed by $\vec{A}$ and $\vec{B}$ as adjacent sides is $\frac{1}{2}\left|\vec{A} \times \vec{B}\right|$.

104.Area of triangle ABC if position vector of A is $\vec{a}$, position vector of B is $\vec{b}$ and position vector of C is $\vec{c}$, then area = $\frac{1}{2} | \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} |$.

105.The area of the parallelogram formed by $\vec{A}$ and $\vec{B}$ as adjacent sides is $\left|\vec{A} \times \vec{B}\right|$.

106.If $\vec{P}$ and $\vec{Q}$ are diagonals of a parallelogram, then area of parallelogram= $\frac{1}{2}\left|(\vec{P} \times \vec{Q})\right|$.

107.Unit vector parallel to $\vec{C}$ or normal to $\vec{A}$ and $\vec{B}$ is $\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$.

108.If $\vec{A}.(\vec{B} \times \vec{C}) = 0$, then $\vec{A}, \vec{B}$ and $\vec{C}$ are coplanar.

109.If $\vec{A} + \vec{B} = \vec{C}$, then $\vec{A}.(\vec{B} \times \vec{C}) = 0$.

110. **Division by a vector:** is not defined because it is not possible to divide a direction by a direction.