

## KINEMATICS

1. The branch of physics that deals with the motion of a body due to the application of force is called **mechanics**.
2. Mechanics is divided into two branches namely dynamics and statics.
3. The branch of mechanics that deals with the state of rest of a body is called **statics**.
4. The branch of mechanics that deals with the state of motion of a body is called **dynamics**.
5. Dynamics is classified into kinematics and kinetics.
6. **Kinematics** is the study of motion which relates to the motion of bodies without reference to either the mass or the force causing it.
7. **Kinetics** is the study of motion which relates to the action of forces causing the motion and the mass that is moved.
8. A body is said to be at rest if its position remains constant with respect to its surroundings or frame of reference.
9. A body is said to be in motion if its position is changing with respect to its surroundings or frame of reference.
10. The line joining the successive positions of a moving body is called its **path**. The length of the path gives the distance travelled by the body.
11. **Displacement** is the directed line segment joining the initial and final positions of a moving body. It is a vector.
12. If every particle of a moving body traverses the same distance along parallel paths, which may be straight or curved, while the body is moving, then the motion of the body is called **translatory motion**.
13. When the path traversed by each particle of a body is a straight line, then its motion is said to be **rectilinear**.
14. When the path traversed by the particles are parallel paths, then the motion is said to be **curvilinear**.

### SPEED :

15. Speed of a body is the rate at which it describes its path. Its SI unit is  $\text{ms}^{-1}$ .
16. Speed is a scalar quantity.
17. 
$$\text{Speed} = \frac{\text{distance travelled}}{\text{time taken}}$$
18. A body is said to be moving with **uniform speed** if it has equal distances in equal intervals of time, however small these intervals may be.
19. A body is said to be moving with **non uniform speed** if it has unequal distances in equal intervals of time or equal distances in unequal intervals of time, however small these intervals may be.
20. 
$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$
21. 
$$\text{Instantaneous speed} = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

22. If a particle covers the 1<sup>st</sup> half of the total distance with a speed 'a' and the second half with a speed 'b'.

$$\text{Average speed} = \frac{2ab}{a+b}.$$

23. If a particle covers 1<sup>st</sup> 1/3<sup>rd</sup> of a distance with a speed 'a', 2<sup>nd</sup> 1/3<sup>rd</sup> of the distance with speed 'b' and 3<sup>rd</sup> 1/3<sup>rd</sup> of the distance with speed 'c'

$$\text{Average speed} = \frac{3abc}{ab + bc + ca}.$$

24.  $1 \text{ kmph} = \frac{5}{18} \text{ ms}^{-1}$ ;  $1 \text{ mph} = \frac{22}{15} \text{ fts}^{-1}$

25. For a body with uniform speed,  
distance travelled = speed x time.

### Velocity :

26. The rate of change of displacement of a body is called **velocity**. Its SI unit is  $\text{ms}^{-1}$ .

27. Velocity is a vector quantity.

28. A body is said to be moving with **uniform velocity**, if it has equal displacements in equal intervals of time, however small these intervals may be.

29. For a body moving with uniform velocity, the displacement is directly proportional to the time interval.

30. If the direction or magnitude or both of the velocity of a body change, then the body is said to be moving with **non-uniform velocity**.

31. **Average velocity** =  $\frac{\text{net displacement}}{\text{total time taken}}$

32. For a body moving with uniform acceleration, the **average velocity** =  $\frac{u+v}{2}$ .

33. The velocity of a particle at any instant of time or at any point of its path is called **instantaneous velocity**.  $\vec{V} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{s}}{\Delta t} = \frac{d\vec{s}}{dt}$

### 34. Average velocity :

- a. If a particle under goes a displacement  $s_1$  along a straight line  $t_1$  and a displacement  $s_2$  in time  $t_2$  in the same direction, then

$$\text{Average velocity} = \frac{s_1 + s_2}{t_1 + t_2}$$

- b. If a particle undergoes a displacement  $s_1$  along a straight line with velocity  $v_1$  and a displacement  $s_2$  with velocity  $v_2$  in the same direction, then

$$\text{Average velocity} = \frac{(s_1 + s_2)v_1v_2}{s_1v_2 + s_2v_1}$$

- c. If a particle travels first half of the displacement along a straight line with velocity  $v_1$  and the next half of the displacement with velocity  $v_2$  in the same direction, then

$$\text{Average velocity} = \frac{2v_1v_2}{v_1 + v_2} \quad (\text{in the case (b) put } s_1 = s_2)$$

- d. If a particle travels for a time  $t_1$  with velocity  $v_1$  and for a time  $t_2$  with velocity  $v_2$  in the same direction, then

$$\text{Average velocity} = \frac{v_1t_1 + v_2t_2}{t_1 + t_2}$$

- e. If a particle travels first half of the time with velocity  $v_1$  and the next half of the time with velocity  $v_2$  in the same direction, then

$$\text{Average velocity} = \frac{v_1 + v_2}{2} \quad (\text{in the case d put } t_1 = t_2)$$

35. Velocity of a particle is uniform if both its magnitude and direction remains unchanged.

36. Velocity of a body changes when magnitude or direction or both change.

37. **Acceleration :**

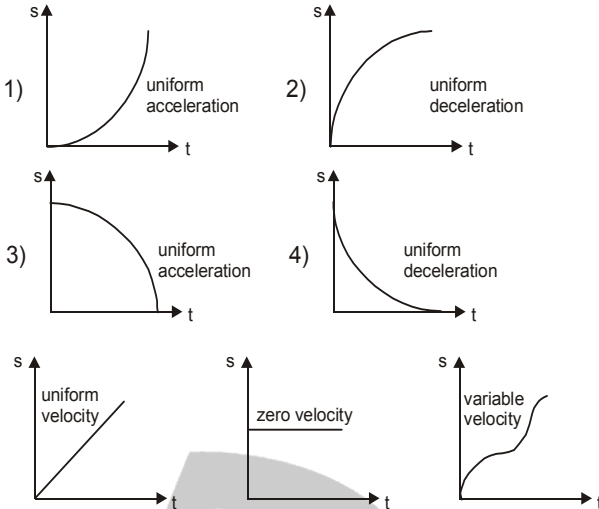
- If the velocity of a body changes either in magnitude or in direction or both, then it is said to have **acceleration**.
- For a freely falling body, the velocity changes in magnitude and hence it has acceleration.
- For a body moving round a circular path with a uniform speed, the velocity changes in direction and hence it has acceleration.
- For a projectile, whose trajectory is a parabola, the velocity changes in magnitude and in direction, and hence it has acceleration.
- The acceleration and velocity of a body need not be in the same direction. eg : A body thrown vertically upwards.
- If equal changes of velocity takes place in equal intervals of time, however small these intervals may be, then the body is said to be in **uniform acceleration**.
- Negative acceleration is called **retardation** or **deceleration**.
- The acceleration of a particle at any instant or at any point is called **instantaneous** acceleration.

$$\bar{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{s}}{dt^2}$$

- For a body moving with uniform acceleration, the average velocity =  $\frac{u+v}{2}$ .
- A body can have zero velocity and non-zero acceleration. Eg : for a particle projected vertically up, velocity at the highest point is zero, but acceleration is  $-g$ .
- If a body has a uniform speed, it may have acceleration. Eg : uniform circular motion
- If a body has uniform velocity, it has no acceleration.
- When a body moves with uniform acceleration along a straight line and has a distance 'x' travelled in the  $n^{\text{th}}$  second, in the next second it travels a distance  $x+a$ , where 'a' is the acceleration.
- Acceleration of free fall in vacuum is uniform and is called acceleration due to gravity ( $g$ ) and it is equal to  $980 \text{ cms}^{-2}$  or  $9.8 \text{ ms}^{-2}$ .

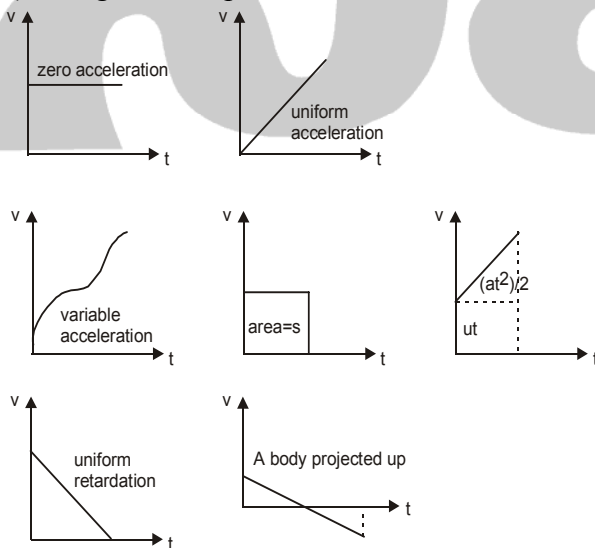
## 38. displacement-time graph :

- 1) Straight lines represents uniform velocity
- 2) Slope of straight line gives velocity
- 3) Smooth curves represents uniform acceleration
- 4) Zig zag curve represents non-uniform acceleration



## 39. Velocity-time graph :

- 1) Slope gives the acceleration.
- 2) Area under the graph gives the distance travelled
- 3) Curve represents non-uniform acceleration.
- 4) Straight line represents uniform acceleration.



## 40. The equations of motion for uniform acceleration :

- 1)  $v = u + at$
- 2)  $s = ut + \frac{1}{2}at^2$
- 3)  $v^2 - u^2 = 2as$
- 4)  $s_n = u + \frac{a}{2}(2n - 1)$
- 5)  $s = \left(\frac{u+v}{2}\right)t$

**One dimensional motion :**

41. If a body starts from rest and moves with uniform acceleration 'a' and if  $D_m$  is the distance travelled by it in  $m^{\text{th}}$  second and  $D_n$  is the distance travelled in  $n^{\text{th}}$  second, then  $a = \frac{D_n - D_m}{n - m}$ .

42. For a particle moving with uniform velocity  $a = 0$   
 $\therefore S = U t$  or  $S \propto t$

43. For a particle moving with uniform retardation along a straight line distance travelled before coming to rest ( $v = 0$ ) is  $s = \frac{u^2}{2a}$

$$\therefore s \propto U^2 \quad \therefore \frac{s_1}{s_2} = \frac{u_1^2}{u_2^2}$$

44. If a particle travels along a straight line with uniform acceleration and travels distances  $S_n$  and  $S_{n+1}$  in two successive seconds, the acceleration of the particle is

$$a = S_{n+1} - S_n$$

45. If a particle travels along a straight line with uniform acceleration and travels distances  $S$  and  $S^I$  in two successive intervals of  $n$  seconds each, the acceleration of the particle is

$$a = \frac{S^I - S}{n^2}$$

46. Moving with uniform acceleration from rest, a body attains a velocity 'v' after a displacement 'x', then its velocity becomes 'nv' after a further displacement  $(n^2 - 1)x$ .

47. If a bullet loses  $(1/n)^{\text{th}}$  of its velocity while passing through a plank, then the no. of such planks required to just stop the bullet is  $= \frac{n^2}{2n-1}$

48. The velocity of a body becomes  $\left(\frac{1}{n}\right)^{\text{th}}$  of its initial velocity after a displacement of 'x', then it will come to rest after a further displacement of  $\frac{x}{n^2 - 1}$ .

49. The displacement of a body is proportional to the square of time, then its initial velocity is zero.

50. Starting from rest a body travels with an acceleration ' $\alpha$ ' for some time and then with deceleration ' $\beta$ ' and finally comes to rest. If the total time of journey is 't', then the maximum velocity and displacement are given by

$$V_{\text{max}} = \frac{(\alpha\beta)t}{\alpha + \beta}, s = \frac{(\alpha\beta)t^2}{2(\alpha + \beta)} \text{ and}$$

$$\text{average velocity} = \left( \frac{V_{\text{max}}}{2} \right)$$

51. If  $R$  is the range of head lights and 'a' is the maximum retardation of an automobile, then its maximum safe speed  $= \sqrt{2aR}$ .

52. If a body starts from rest and moves with uniform acceleration, then the ratio of the times to cover  $1^{\text{st}}, 2^{\text{nd}}, 3^{\text{rd}}, 4^{\text{th}} \dots \dots n^{\text{th}}$  metres of the distance is  $(\sqrt{1} - \sqrt{0}) : (\sqrt{2} - \sqrt{1}) : (\sqrt{3} - \sqrt{2}) \dots (\sqrt{n} - \sqrt{n-1})$

53. A body is projected vertically up from a topleless car relative to the car which is moving horizontally relative to earth.

a. If car velocity is constant, ball will be caught by the thrower.

- b. If car velocity is constant, path of ball relative to the ground is a parabola and relative to this car is straight up and then straight down.
- c. If the car accelerates, ball falls back relative to the car.
- d. If acceleration or retardation of the car is constant path relative to car is a straight line and relative to ground is a parabola.

54. The equations of motion for a body

- a) projected up                      b) freely falling

$$v = u - gt$$

$$v = gt$$

$$s = ut - \frac{1}{2}gt^2$$

$$s = \frac{1}{2}gt^2$$

$$v^2 - u^2 = -2gs$$

$$s_n = \frac{g}{2}(2n - 1)$$

$$\text{time of rise} = \frac{u}{g}$$

$$\text{time of fall} = \frac{u}{g} = \sqrt{\frac{2h}{g}}$$

55. **Freely falling body :**

- a. Any freely falling body travels  $g/2$  metres or 4.9 m in the first second.
- b. During the free fall of a body, after a certain interval of time, if gravity disappears then the body continues to move with uniform velocity acquired during its free fall.
- c. The displacements of a freely falling body in successive seconds or in equal intervals of time are in the ratio of 1 : 3 : 5 : 7 : ..... This also holds good for a body starting from rest and moving with uniform acceleration.
- d. The displacements of a freely falling body in 1 s, 2 s, 3 s, 4 s, ..... n s are in the ratio  $1^2 : 2^2 : 3^2 : 4^2 : 5^2 : 6^2 : \dots$
- e. In the presence of air resistance, the acceleration of a denser body is greater.
- f. The acceleration of a body in a medium is given by  $g^1 = g \left( \frac{1 - d_m}{d_b} \right) = g \left( 1 - \frac{d_m}{d_b} \right)$

( $d_m$  = density of the medium)

( $d_b$  = density of the body)

If  $d_m = d_b$ ;  $g^1 = 0$ . So it will remain at rest or in uniform motion.

- g. A freely falling body passes through two points A and B in time intervals of  $t_1$  and  $t_2$  from the start, then the distance between the two points is given by  $AB = \frac{g}{2}(t_2^2 - t_1^2)$
- h. A freely falling body passes through two points A and B distant  $h_1$  and  $h_2$  from the start, then the time taken by it to move from A to B is given by  $T = \sqrt{\frac{2h_2}{g}} - \sqrt{\frac{2h_1}{g}} = \sqrt{\frac{2}{g}}(\sqrt{h_2} - \sqrt{h_1})$
- i. Two bodies are dropped from heights  $h_1$  and  $h_2$  simultaneously. Then after any time the distance between them is equal to  $(h_2 - h_1)$
- j. stone is dropped into a well of depth 'h', then the sound of splash is heard after a time of 't' given by  $t = \sqrt{\frac{2h}{g}} + \frac{h}{v_{\text{sound}}}$
- k. A stone is dropped into a river from the bridge and after 'x' second another stone is projected down into the river from the same point with a velocity of 'u'. If both the stones reach the water simultaneously, then

$$\frac{1}{2}gt^2 = u(t-x) + \frac{1}{2}g(t-x)^2$$

**Body thrown vertically upwards :**

56. The time of flight of a body thrown up vertically with velocity  $u$  is  $2u/g$ .

$$\text{maximum height} = \frac{u^2}{2g}$$

57.  $H_{\max} = \frac{u^2}{2g} \Rightarrow H_{\max} \propto u^2$  (independent of the mass of the body)  $\Rightarrow \frac{H_1}{H_2} = \frac{u_1^2}{u_2^2} = \left(\frac{u_1}{u_2}\right)^2$

58. If a body projected vertically upwards takes 't' seconds to reach the maximum height, then in the first  $t/2$  seconds it travels  $3/4^{\text{th}}$  of the maximum height.

59.  $t_a = t_d = \frac{u}{g} \Rightarrow T = t_a + t_d = \frac{2u}{g} \Rightarrow T \text{ or } t_a \propto u$

60. At any point of the journey, a body possesses the same speed while moving up and while moving down.

61. The height reached in the first second of ascent is equal to the height of fall in the last second of descent.

62. Irrespective of velocity of projection, all the bodies pass through a height  $\frac{g}{2}$  in the last second of ascent.

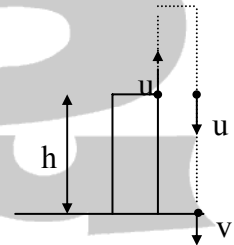
63. The change in velocity over the complete journey is '2u' (downwards).

64. If a vertically projected body rises through a height 'h' in  $n^{\text{th}}$  s, then in  $(n-1)^{\text{th}}$  s it will rise through a height  $h + g$  and in  $(n+1)^{\text{th}}$  s it will rise through height  $h - g$ .

65. If a body is projected vertically up from the top a tower of height  $h$  with a velocity  $u$  and takes "t" seconds to reach the ground

$$h = -u t + \frac{1}{2}gt^2 \Rightarrow h = \frac{1}{2}gt^2 - ut;$$

$$h = \frac{v^2 - u^2}{2g}$$



66. If an object is dropped from a balloon rising up with a velocity  $u$  at a height  $h$ .

a) Equation of motion relative to earth is

$$h = \frac{1}{2}gt^2 - ut$$

b) Equation of motion relative to balloon is

$$h = \frac{1}{2}(g + a_B)t^2, \text{ where } a_B \text{ is acceleration of the balloon.}$$

c) Relative to earth body goes up and then falls

d) Relative to the balloon it falls vertically downward.

67. If a body is projected vertically up with a velocity  $u$  from a tower and it reaches the ground with a velocity  $nu$ , the height of the tower is

$$h = \frac{u^2}{2g}(n^2 - 1)$$

68. A body is thrown vertically upwards with a velocity 'u' from the ground reaches a point 'P' on its path at height 'h' after a time  $t_1$  and  $t_2$  from the beginning, then

$$u = \frac{g}{2}(t_1 + t_2); \quad h = \frac{1}{2}gt_1t_2.$$

69. A particle projected vertically up from the top of a tower takes  $t_1$  sec to reach the ground. Another particle thrown downwards with the same velocity from the top of the tower takes  $t_2$  seconds to reach the ground. If the particle is dropped from the top of the tower, time taken is  $t$ , then

a)  $t = \sqrt{t_1t_2}$

b) height of the tower is  $h = \frac{1}{2}gt_1t_2$

c) Velocity of projection is  $u = \frac{g}{2}(t_1 - t_2)$

d) In the first and second case body reaches the ground with the same velocity

70. A body is dropped from the top edge of a tower of height 'h' and at the same time another body is projected vertically up from the foot of the tower with a velocity 'u', then they will meet after a time of

$$t = \frac{h}{u} \text{ and at a distance of}$$

$$h_1 = \frac{1}{2}gt^2 = \frac{gh^2}{2u^2} \text{ from the top of the tower (or)}$$

$$h_2 = h - \frac{gh^2}{2u^2} \text{ from the foot of the tower.}$$

Their velocities at the meeting point are

$$V_1 = gt = \frac{gh}{u} \text{ (freely falling body)}$$

$$V_2 = u - \frac{gh}{u} \text{ (vertically projected body)}$$

71. In the absence of air resistance, time of ascent and time of descent are equal.

72. Time of rise < time of fall, if air resistance is taken into account.

73. A elevator is accelerating upwards with an acceleration  $a$ . If a person inside the elevator throws a particle vertically up with a velocity  $u$  relative to the elevator, time of flight is  $t = \frac{2u}{g+a}$

74. In the above case if elevator accelerates down, time of flight is  $t = \frac{2u}{g-a}$

### PROJECTILES :

75. A body thrown with an angle with the horizontal is called a **projectile**.

76. The path traced by a projectile is called **trajectory** and is a parabola.

77. **Oblique projectile :**

1) For a projectile, the horizontal component of velocity is  $(u\cos\theta)$ . It remains constant throughout the motion

2) The vertical component  $(u\sin\theta)$  is subjected to acceleration due to gravity.



78. **Equations for an oblique projectile :**

a) Maximum height reached =  $\frac{u^2 \sin^2 \theta}{2g}$

b) Time of flight =  $\frac{2u \sin \theta}{g}$

Time of rise = time of fall =  $\frac{u \sin \theta}{g}$

c) Range =  $\frac{u^2 \sin 2\theta}{g}$ ;  $R_{\max} = \frac{u^2}{g}$  for  $\theta = 45^\circ$

d)  $\tan \theta = \frac{4H_{\max}}{R}$ ;  $\tan \theta = \frac{gT^2}{2R}$

79. **At the maximum height**

a) Kinetic Energy =  $\frac{1}{2}mu_x^2 = \frac{1}{2}mu^2 \cos^2 \theta$

b) Potential Energy =  $mgH_{\max} = \frac{1}{2}mu^2 \sin^2 \theta$

c) Total Energy = K.E + P.E =  $\frac{1}{2}mu^2$

d) If K.E = P.E then  $\theta = 45^\circ$

e)  $\tan^2 \theta = \frac{P.E}{K.E}$

80. **Velocity after time "t":**

a)  $a_x = 0$ ;  $a_y = -g$

b) Horizontal component of velocity through out the motion is constant.

c) vertical component of velocity changes with time

d) Horizontal component of velocity  $V_x = U \cos \theta$

e) Vertical component of velocity  $V_y = U \sin \theta - gt$

f) Velocity of the particle  $V = \sqrt{V_x^2 + V_y^2}$

g) Velocity of a projectile after t seconds

$$v = \sqrt{(u \cos \theta)^2 + (u \sin \theta - gt)^2}$$

h) Velocity of a projectile when it is at a height h is

$$v = \sqrt{(u \cos \theta)^2 + [(u \sin \theta)^2 - 2gh]} \text{ or } \sqrt{u^2 - 2gh}$$

i) Direction of motion w. r. t. to horizontal.

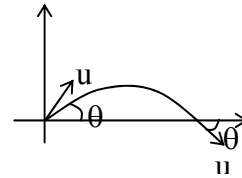
$$\alpha = \tan^{-1} \frac{V_y}{V_x}$$

j) If  $\alpha$  is the angle made by a projectile after t seconds, then  $\tan \alpha = \frac{u \sin \theta - gt}{u \cos \theta}$

k) If  $\alpha$  is the angle made by a projectile after traveling a height of h, then

$$\tan \alpha = \frac{\sqrt{(u \sin \theta)^2 - 2gh}}{u \cos \theta}$$

- l) Velocity at highest point is  $U\cos\theta$  in the horizontal direction.  
 m) Vertical component of velocity at the highest point  $V_y = 0$   
 n) Velocity and acceleration are perpendicular to each other at the highest point.



- o) If projected from level ground velocity of landing and angle of landing is same in magnitude as during projection.  
 p) If projected from level ground velocity is maximum during projection and during landing and minimum at the highest point.  
 q) **If projected from level ground**  
 a) Change in velocity till it reaches highest point =  $U\sin\theta$   
 b) Change in velocity for complete trajectory =  $2U\sin\theta$   
 r) Velocity of the projectile when it moves perpendicular to its initial velocity is  $U\cot\theta$ .  
 s) Time taken for the velocity to become perpendicular to the initial velocity is  $\frac{u}{g\sin\theta}$   
 t) If  $V_1$  and  $V_2$  are the magnitudes of velocities at heights  $h_1$  and  $h_2$ ;  $V_2^2 - V_1^2 = -2g(h_2 - h_1)$

### 81. Position of the projectile after time 't':

- a) If  $x$  and  $y$  represent the horizontal and vertical displacements with respect the point of projection 't' seconds after projection

$$x = (U \cos\theta) t$$

$$y = (U \sin\theta) t - \frac{1}{2}gt^2$$

- b) Equation of trajectory is

$$Y = (\tan\theta) x - \frac{1}{2}g \frac{x^2}{u^2 \cos^2 \theta}$$

$$= \tan\theta x - \frac{g}{2u^2 \cos^2 \theta} \cdot x^2$$

- c)  $y = Ax - Bx^2$ ;  $A = \tan\theta$ ,  $B = \frac{g}{2u^2 \cos^2 \theta}$

$$H_{\max} = \frac{A^2}{4B}; \quad \text{Range } R = \frac{A}{B}$$

$$\text{and } \frac{H_{\max}}{R} = \frac{A}{4} \quad \text{or } \tan\theta = \frac{4H_{\max}}{R}$$

### 82. Complementary angles of projection

- a) For a given velocity of projection angles of projection are  $\theta$  and  $90-\theta$  then they are called as complementary angles of projection.  
 Ex:  $30^\circ$ ,  $60^\circ$   
 b) Range is equal for complementary angles of projection ( $u = \text{constant}$ )  $R = \text{Constant}$  for  $\theta$  and  $90-\theta$ .  
 c) If  $h_1$  and  $h_2$  are the maximum heights attained for complementary angles of projection

$$h_1 + h_2 = \frac{u^2}{2g}; \quad R = 4\sqrt{h_1 h_2}$$

$$\tan^2 \theta = \frac{h_1}{h_2}; \quad R_{\max} = 2(h_1 + h_2)$$

d) If  $t_1$  and  $t_2$  are the times of flight for complementary angles of projection

$$R = \frac{1}{2}gt_1t_2$$

83. If a man throws a body to a maximum distance  $R$  then he can project the body to vertical height  $R/2$ .

84. If a man throws a body to a maximum distance  $R$  then the greatest height attained by the body is  $R/4$ .

a) The angle between velocity and acceleration during the rise of projectile is  $180^\circ < \theta < 90^\circ$ .

b) The angle between velocity and acceleration during the fall of projectile is  $0^\circ < \theta < 90^\circ$

85. If a body is projected up at an angle  $\theta$  with the horizontal from the top of a tower of height 'h', then

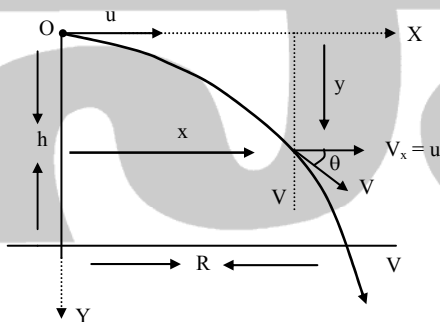
$$h = -(u \sin \theta) t + \frac{1}{2}gt^2$$

86. If a body is projected down at an angle  $\theta$  with the horizontal from the top of a tower then

$$h = -(u \sin \theta) t + \frac{1}{2}gt^2$$

### Horizontal projection :

87. When a body is projected, horizontally from the top of a tower path of the body is parabola relative to ground.



a) it reaches the ground tracing a parabolic path.

b) its time of descent is  $\sqrt{2h/g}$ .

c) during its journey it suffers a horizontal displacement of  $u\sqrt{2h/g}$ .

d) the angle  $\alpha$  with which it strikes the ground is given by

$$\tan \alpha = \frac{\sqrt{2h/g}}{u} = \frac{gt}{u}$$

e) the velocity with which it hits the ground is given

$$\text{by } v = \sqrt{u^2 + 2gh} \text{ or } v = \sqrt{u^2 + (gt)^2}.$$

88. If a body is projected horizontally and another is dropped from the same height, both the bodies will take same time to reach the ground.

### Position after time t :

Horizontal displacement after time t.  $x = u t$

Distance fallen in time "t"

$$y = \frac{1}{2}gt^2$$

**Velocity after time t :**

$$v = \sqrt{u^2 + (gt)^2} = \sqrt{u^2 + 2gh}$$

$$\text{If angle made with the horizontal is } \alpha \quad \tan \alpha = \frac{gt}{u} = \frac{gt}{\sqrt{2gh}}$$

**Equation of path:**

$$y = \frac{1}{2}g \frac{x^2}{u^2}$$

89. From a certain height. If two bodies are projected horizontally with velocities  $u_1$  and  $u_2$  in opposite directions.

a) Time after which velocity vectors are perpendicular is  $t = \frac{\sqrt{u_1 u_2}}{g}$

b) Time after which displacement vectors are perpendicular is  $t = \frac{2\sqrt{u_1 u_2}}{g}$

- c) Distance between the two bodies when velocity vectors are perpendicular is

$$\frac{\sqrt{u_1 u_2}}{g} (u_1 + u_2)$$

- d) Horizontal distance between the two bodies when displacement vectors are perpendicular is

$$2 \frac{\sqrt{u_1 u_2}}{g} (u_1 + u_2)$$

90. Two bodies are projected from a tower horizontally with velocities ' $u_1$ ' and ' $u_2$ ' then

$$\frac{t_1}{t_2} = 1 \text{ and } \frac{x_1}{x_2} = \frac{u_1}{u_2} \text{ and } \frac{v_1}{v_2} = \sqrt{\frac{u_1^2 + 2gh}{u_2^2 + 2gh}}$$

91. A bomb dropped from a plane moving horizontally with a uniform velocity reaches the ground following a parabolic path. (As seen by the pilot the bomb takes a vertically downward path).
92. From the top of a tower a stone is dropped and simultaneously another stone is projected horizontally with a uniform velocity. Both of them reach the ground simultaneously.
93. In the case of a projectile, velocity varies both in magnitude and direction but the acceleration remains constant both in magnitude and direction.
94. If air resistance is considered, trajectory departs from parabola; time of flight increases; striking angle increases; range decreases; maximum height decreases; striking velocity decreases and the time of ascent is less than the time of descent.
95. For projectiles like Inter Continental Ballistic Missiles (ICBM) the trajectory is a portion of an ellipse (due to large variation in altitude).

**Motion of a body along an inclined plane :**

96. Acceleration of a body sliding down a smooth inclined plane of angle of inclination  $\theta$  with the horizontal is given by  $a = g \sin \theta$

97. If a body travels from rest on a smooth inclined plane

$$\text{Velocity } v = \sqrt{2gl \sin \theta} = \sqrt{2gh}$$

$$\text{Time taken } t = \sqrt{\frac{2l}{g \sin \theta}} = \sqrt{\frac{2h}{g \sin^2 \theta}}.$$

98. A body on an inclined plane reaches the bottom with the same velocity as that of a freely falling body but in a different direction after a time of  $\frac{1}{\sin \theta}$  times that of a freely falling body.

www.ck12.com