

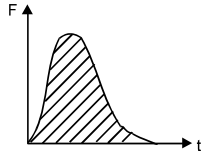
## DYNAMICS

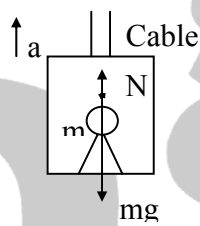
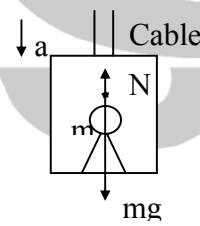
*Synopsis : NEWTON'S LAWS OF MOTION :*

1. **Momentum** is the quantity of motion possessed by a body by virtue of which it can set other bodies in motion by collision.
2. Momentum is the product of mass and velocity ( $\vec{P} = m\vec{v}$ ). SI unit is  $\text{kg ms}^{-1}$ . It is a vector having the same direction as that of velocity.
3. In finding the change in momentum, vector subtraction must be used.
4. If a ball of mass  $m$  moving with a speed  $v$  strikes a wall at right angle to it and rebounds with the same speed, then the change in momentum is  $2mv$ .
5. If a body of mass  $m$  thrown vertically upwards with a velocity  $u$  returns to the starting point, then the change in its momentum is  $2mu$ .
6. If a ball of mass  $m$  moving with a velocity  $u$  is struck by a bat and retraces its path with a velocity  $v$ , then the change in momentum is  $m(v + u)$ .
7. **Newton's first law of motion** : Every body continues to be in the state of rest or of uniform motion unless it is compelled by an external force to change that state. i.e., the momentum of a body remains constant as long as no external force acts on it.
8. The first law of motion leads to the concepts of **force** and **inertia**.
9. **Inertia** is the tendency of a body to preserve its state of rest or of uniform motion along a straight line in the absence of any external force
10. The three types of inertia are:
  - i) inertia of rest
  - ii) inertia of motion and
  - iii) Inertia of direction
11. **Inertia of rest**: The inability of a body to change its state of rest by itself is called inertia of rest. Eg: When a bus at rest starts suddenly passengers fall back
12. **Inertia of motion**: the inability of a body to change its uniform motion by itself is called as inertia of motion. Eg: when a bus in uniform motion suddenly stops, the passengers fall forward.
13. **Inertia of direction**: The inability of a body to change its direction of motion by itself is called inertia of direction. Eg: When a bus takes a turn passengers will be pressed outwards.
14. **Force** is that which changes or tends to change the state of rest or of uniform motion of a body along a straight line.
15. **Newton's second law of motion** : The rate of change of momentum of a body is directly proportional to the impressed force and takes place in the direction of force.
16. The second law of motion gives the direction and magnitude of force.
17. Force =  $\frac{\text{change in momentum}}{\text{time}}$ , i.e.,  $\vec{F} = \frac{m(\vec{v} - \vec{u})}{t}$ .
18. Force = mass x acceleration ;  $\vec{F} = m\vec{a}$
19.  $F = \frac{mv - mu}{t}$        $F = \frac{m(v - u)}{t}$   
 $F = m \frac{dv}{dt}$        $F = v \cdot \frac{dm}{dt}$   
 $F = ma$        $F = m \left( \frac{v^2 - u^2}{2s} \right)$   
 $F = \frac{m \times 2(s - ut)}{t^2}$        $F = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$$F = \frac{\text{work}}{s} \quad F = \frac{\text{power}}{\text{velocity}}$$

$$F = \text{pressure} \times \text{area} \quad F = mg$$

20. **A unit force** : is one which when acting on unit mass produces unit acceleration in its direction.  
Units : SI unit is newton and cgs unit is dyne;  $1 \text{ N} = 10^5 \text{ dynes}$ .
21. Gravitational unit of force :  
 $1 \text{ kgwt} = g \text{ N} = 9.8 \text{ newton}$   
 $1 \text{ gwt} = g \text{ dynes} = 980 \text{ dynes}$ .  
 Force = rate of change of mass x change in velocity  
 $F = \frac{m(v-u)}{t}$  [rocket, conveyor belt problems, etc.  
 can be solved by this formula]
22. If a rocket ejects the exhaust gases with a velocity  $u$  relative to the rocket at the rate of  $\frac{dm}{dt}$ , the force  $F$  acting on the rocket is  $F = u \left( \frac{dm}{dt} \right)$ .
23. If gravel is dropped on a conveyor belt at the rate of  $\frac{dm}{dt}$ , the extra force required to keep the belt moving with velocity is  $F = u \left( \frac{dm}{dt} \right)$ .
24. A jet of water of density  $d$  from a tube of area of cross section  $a$  comes out with a velocity  $v$ .  
 a) Average force exerted by tube on water is  $dAv^2$   
 b) Force required to hold the tube in a fixed position =  $dAv^2$   
 c) If the water traveling horizontally strikes a vertical wall normally and then flows down along the wall, the normal force exerted on the wall is  $dAv^2$ .  
 d) In the above case if water rebounds with the same speed, force exerted on the wall is  $2dAv^2$   
 e) In the above case if water strikes the surface at angle  $\theta$  with the normal and reflects with the same speed, force exerted on the wall is  $2dAv^2 \cos\theta$ .
25. If a gun fires  $n$  bullets each of mass  $m$  per second each with a velocity  $u$ , the force  $F$  necessary to hold the gun is  $F = mnu$ .
26. A very large force acting for a short interval of time is called **impulsive force**. Eg : Blow of a hammer on the head of a nail.
27. The impulse of a force is defined as the product of the average force and the time interval for which it acts.  
 Impulse  $J = F_{AV} \Delta t = m \bar{v} - m \bar{u}$   
 Impulse momentum theorem =  $\int_{t_1}^{t_2} \vec{F} dt = m \bar{v} - m \bar{u}$
28. Impulse due to a variable force is given by the area under  $F-t$  graph.  
 Impulse =  $\int_{t_1}^{t_2} \vec{F} dt$
- 
29. While catching a fast moving cricket ball the hands are lowered, there by increasing the time of catch which thus decreases the force on hands.
30. A person jumping on to sand experiences less force than a person jumping on to a hard floor, because sand stops the person in more time.

31. If a force  $F_1$  acts on a body at rest for a time  $t_1$  and after that another force  $F_2$  brings it to rest again in a time  $t_2$ , then  $F_1 t_1 = F_2 t_2$ .
32. The gravitational force that acts on a body is called its **weight** ( $W = mg$ ). It is a vector always pointing in a vertically downward direction.
33. A bird is in a wire cage hanging from a spring balance when the bird starts flying in the cage, the reading of the balance decreases.
34. In the above case, if the bird is in a closed cage or air - tight cage and it hovers in the cage, the reading of the spring balance does not change.
35. In the above case for a closed cage if the bird accelerates upward reading of the balance is  $R = W_{\text{bird}} + ma$ , where  $m$  is the mass of the bird and  $a$  its acceleration.
36. If a block of mass  $m$  hangs at the end of a massless string and the string is pulled up, the tension in string is  $T = m(g + a)$  if the block accelerates in the upward direction.  $T = m(g - a)$  if the block is accelerated in the downward direction.  $T = mg$  if the block is moved up or down with uniform speed.
37. When a man stands on a weighing machine, the weighing machine measures the normal force between the man and the machine.
38. When the man and the weighing machine are at rest relative to the earth, reading of the weighing machine is  $N = mg = \text{Weight of the man}$ .  $N$  is also called **apparent weight**.
39. **Man inside an elevator:**
- a) Elevator accelerates up:  
 Relative to earth  $N - mg = ma$   
 ( $M = \text{mass of man}$ )  
 Apparent weight  $= N = m(g + a)$   
 Tension in the cable  
 $T = (M_{\text{elevator}} + M_{\text{man}})(g + a)$   
 Same effect is felt when elevator retards while going down.
- b) Elevator accelerates downward:  
 Relative to earth  $mg - N = ma$   
 Apparent weight  $= N = m(g - a)$   
 Tension in the cable  
 $T = (M_{\text{elevator}} + M_{\text{man}})(g - a)$   
 Same effect is felt when elevator goes up and retards.  
 If elevator falls freely (cable breaks)  $N = 0$   
 i.e. apparent weight of in a free fall = 0
- 
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40. Man inside an artificial satellite
- a) An artificial satellite orbiting the earth in a circular orbit is a freely falling body because its centripetal acceleration is equal to the acceleration due to gravity in that orbit.
41. With a car at rest, mark the position of the stationary pendulum bob on the table under it, with the car in motion
- a) If the bob remains over the mark only when the car is moving in a straight line at a constant speed. (inertial frame of reference)
- b) If the car is gaining or losing the speed or is negotiating a bend, the bob moves from its mark and the car is a non-inertial frame.
42. If you put a ball at rest on a rotating merry-go round, no identifiable force acts on the ball but it does not remain at rest, it is non - inertial reference frame.
43. **Newton's third law :** For every action there is an equal and opposite reaction.

44. Newton's first and third laws are only special cases of second law.
45. **Thrust** is the total force applied on a given area. It is measured as the product of pressure and the area on which the pressure is applied.
46. In nature forces always occur in pairs (action and reaction)
47. When a body exerts a force on another body, the second body exerts a force on the first body of the same magnitude but in opposite direction.
48. If we tie one end of a string to any point of a body and pull at the other end of the string, we exert a force on the body. Such a force exerted by means of a string is called **tension**.
49. When two objects are connected by an inextensible massless string passing over a smooth pulley or peg, then (i) both will have the same acceleration and ii) the tension is the same on both the sides of the pulley.

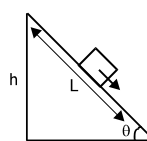
50. **Motion on a smooth inclined plane :**

- (i)  $a = g \sin \theta$   
 (ii) final velocity at the bottom of the inclined plane

$$v = \sqrt{2gL \sin \theta} = \sqrt{2gh}$$

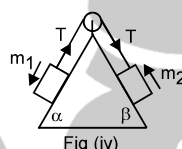
- (iii) time taken to reach the bottom

$$t = \sqrt{\frac{2L}{g \sin \theta}} = \frac{1}{\sin \theta} \sqrt{\frac{2h}{g}}$$



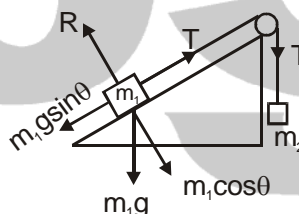
51. acceleration (a) =  $\frac{g(m_1 \sin \alpha - m_2 \sin \beta)}{m_1 + m_2}$

tension =  $T = \frac{m_1 m_2 (\sin \alpha + \sin \beta) g}{m_1 + m_2}$



52. Two masses  $m_1$  and  $m_2$  connected by a string pass over a pulley.  $m_2$  is suspended and  $m_1$  slides up over a frictionless inclined plane of angle  $\theta$

a) Acceleration  $a = \frac{(m_2 - m_1 \sin \theta) g}{m_1 + m_2}$



- b) Tension in the string

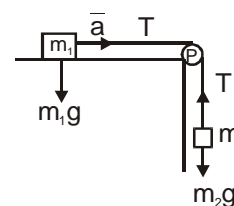
$$T = m_2 g - m_2 a = \frac{m_1 m_2 [1 + \sin \theta] g}{m_1 + m_2}$$

53. A body of mass  $m_1$  is placed on a smooth table. A string attached to  $m_1$  passes over a light pulley and carries a mass  $m_2$ . Acceleration of the system.

i)  $a = \frac{m_2 g}{m_1 + m_2}$

ii) Tension in the string  $T = \left( \frac{m_1 m_2}{m_1 + m_2} \right) g$

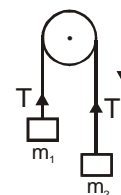
iii) Thrust on the pulley  $P = \sqrt{2} T$



**MOTION OF CONNECTED BODIES :**

54. When two bodies are connected by a light string passing over a frictionless pulley
- a)  $m_1$  and  $m_2$  will have the same acceleration 'a' If  $m_2 > m_1$

$$a = \frac{(m_2 - m_1) g}{m_1 + m_2}$$

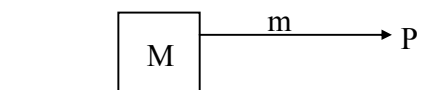


b) The tension is same on both sides of pulley  $T = \frac{2m_1m_2g}{m_1 + m_2}$

c) Thrust (p) on the pulley is  $2T$

$$P = \frac{4m_1m_2g}{m_1 + m_2}$$

55. A block of mass  $M$  is pulled by a rope of mass  $m$  by a force  $P$  on a smooth horizontal plane.

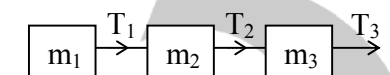


a) Acceleration of the block  $a = \frac{P}{M+m}$

b) Force exerted by the rope on the block

$$F = \frac{Mp}{(M+m)}$$

56. Masses  $m_1, m_2, m_3$  are inter connected by light string and are pulled with a string with tension  $T_3$  on a smooth table.



a) Acceleration of the system  $a = \frac{T_3}{(m_1 + m_2 + m_3)}$

b) Tension in the string

$$T_1 = m_1 a = \frac{m_1 T_3}{m_1 + m_2 + m_3}$$

$$T_2 = (m_1 + m_2) a = \frac{(m_1 + m_2) T_3}{m_1 + m_2 + m_3}$$

$$T_3 = (m_1 + m_2 + m_3) a$$

57.

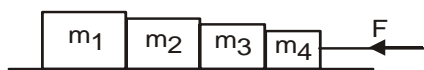


Fig (vi)

Acceleration of the system

$$(a) = \frac{F}{(m_1 + m_2 + m_3 + m_4)}$$

Contact force between  $m_1$  and  $m_2$ ,

$$F_{12} = \frac{m_1 F}{(m_1 + m_2 + m_3 + m_4)}$$

Contact force between  $m_2$  and  $m_3$ ,

$$F_{23} = \frac{(m_1 + m_2) F}{(m_1 + m_2 + m_3 + m_4)}$$

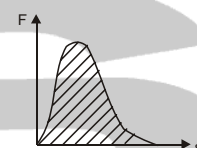
Contact force between  $m_3$  and  $m_4$ ,

$$F_{34} = \frac{(m_1 + m_2 + m_3) F}{(m_1 + m_2 + m_3 + m_4)}$$


58. A rope of length 'L' is pulled by a constant force 'F'. The tension in the rope at a distance 'x' from the end where it is applied is  $F(1 - \frac{x}{L})$ .
59. Limitations of Newton's law of motion:
- It is applicable only for speeds  $V \ll C$  (C = speed of light)
  - It is not applicable in the domain of atoms, molecules, sub atomic particles.
  - It is not applicable when there is a very strong gravitational field.
  - The concept of Newton III law is not applicable, when particles interact with each other by means of a force field.
  - Newton's laws are not applicable for very small accelerations. ( $a < 10^{-10} \text{ ms}^{-2}$ )

**WORK-POWER-ENERGY :**

- Work** is said to be done when the point of application of force has some displacement in the direction of the force.
- The amount of work done is given by the dot product of force and displacement.  
 $\vec{F} \cdot \vec{s} = F s \cos \theta$
- Work is independent of the time taken and is a scalar.
- If the force and displacement are perpendicular to each other, then the work done is zero.
- A person rowing a boat upstream is at rest with respect to an observer on the shore. According to the observer the person does not perform any work. However, the person performs work against the flow of water. If he stops rowing the boat, the boat moves in the direction of flow of water and work is performed by the force due to flow, as there is displacement in the direction of flow.
- If the work is done by a uniformly varying force such as restoring force in a spring, then the work done is equal to the product of average force and displacement.
- If the force is varying non-uniformly, then the work done  $= \int \vec{F} \cdot d\vec{s} = \int F \cdot ds \cdot \cos \theta$ .
- The area of F-s graph gives the work done.
- SI unit of work is **joule**. Joule is the work done when a force of one newton displaces a body through one metre in the direction of force.
- CGS unit of work is erg;  $1 \text{ J} = 10^7 \text{ ergs}$ .
- The work done in lifting an object of mass m through a height 'h' is equal to mgh.
- When a body of mass m is raised from a height  $h_1$  to height  $h_2$ , then the work done  $= mg(h_2 - h_1)$ .
- Let a body be lifted through a height 'h' vertically upwards by a force 'F' acting upwards. Then, the work done by the resultant force is  $W = (F - mg)h$ .
- The work done on a spring in stretching or compressing it through a distance x is given  $W = \frac{1}{2} kx^2$  where k is the force constant or spring constant.
- Work done in changing the elongation of a spring from  $x_1$  to  $x_2$  is  $W = \frac{1}{2} k(x_2^2 - x_1^2)$ .
- a) The work done in pulling the bob of a simple pendulum of length L through an angle  $\theta$  as shown in the figure is  
 $W = mgL(1 - \cos \theta) = 2mgL \sin^2(\theta/2)$   
b) the velocity acquired by it when released from that position is  $v = \sqrt{2gl(1 - \cos \theta)}$



17. The work done in lifting a homogeneous metal rod lying on the ground such that it makes an angle 'θ' with the horizontal, is  $W = \frac{mgl \sin \theta}{2}$ .

18. The work done in rotating a rod or bar of mass m through an angle θ about a point of suspension is  $W = \frac{mgl}{2}(1 - \cos \theta) = mgL \sin^2(\theta/2)$  where L is the distance of the centre of gravity from the point of suspension. 

19. The work done in lifting a body of mass 'm' and density 'd<sub>s</sub>' in a liquid of density 'd<sub>l</sub>' through a height 'h' under gravity is

$$W = m g h \left( 1 - \frac{d_l}{d_s} \right)$$

20. Work done in pulling back a  $\frac{1}{n}$  part of length of a chain hanging from the edge onto a smooth horizontal table completely is  $W = \frac{mgl}{2n^2}$ .

21. Inclined plane :

i) Work done in moving a block of mass 'm' up a smooth inclined plane of inclination 'θ' through a distance 's' is

$$W = Fs = mg \sin \theta s$$

ii) if the plane is rough, then  $W = mg (\sin \theta + \mu_k \cos \theta) s$

22. Work done by a gas during expansion at constant pressure 'P' is given by

Work done = (pressure) (change in volume)

$$W = P (dv) = P (V_2 - V_1)$$

Note: The above formula can also be used to calculate the work done by the heart in pumping the blood.

i) If pressure also varies, then  $W = \int_{V_1}^{V_2} P dV$

Work is positive if  $V_2 > V_1$  i.e. when gas expands and negative if  $V_2 < V_1$  i.e. when gas is compressed.

ii) Area of P - V graph gives work done by the gas.

23. Rate of doing work is called **power**.

$$\text{Power} = \frac{\text{work}}{\text{time}} = \text{Force} \times \text{velocity}.$$

24. SI unit of power is **watt** and CGS unit is **erg/second**.

25. One horse power = 746 watt.

26. If a vehicle travels with a speed of v overcoming a total resistance of F, then the power of the engine is given by  $P = \vec{F} \cdot \vec{v}$ .

27. If a body is rotated in circular path, the power exerted is given by

$$P = \tau \frac{d\theta}{dt} = \tau \omega$$

28. If a block of mass 'm' is pulled along the smooth inclined plane of angle 'θ', with constant velocity 'v', then the power exerted is,  $p = (mg \sin \theta)v$

29. If the block is pulled up a rough inclined plane then the power is  $P = mg (\sin \theta + \mu_k \cos \theta)v$

30. If the block is pulled down a rough inclined plane then the power is  $P = mg (\sin \theta - \mu_k \cos \theta)v$

31. When water is coming out from a hose pipe of area of cross section 'A' with a velocity 'v' and hits a wall normally and

- i) stops dead, then force exerted by the water on the wall is  $Av^2 \rho$ . And the power exerted by water is  $P = A v^3 \rho$  ( $\rho =$  density of water)
- ii) If water rebounds with same velocity ( $v$ ) after striking the wall,  $P = 2Av^3 \rho$
32. When sand drops from a stationary dropper at a rate of  $\frac{dm}{dt}$  on to a conveyer belt moving with a constant velocity, then the extra force required to keep the belt moving with a constant speed  $V$  is given by  $F = v \cdot \frac{dm}{dt}$
- and the power required =  $P = \frac{dm}{dt} v^2$
33. If a pump lifts the water from a well of depth 'h' and imparts some velocity 'v' to the water, then the power of pump
- $$P = \frac{mgh + \frac{1}{2}mv^2}{t}$$
34. Power exerted by a machine gun which fires 'n' bullets in time 't' is  $P = \frac{W}{t} = \frac{n \left( \frac{1}{2}mv^2 \right)}{1}$
- $$P = \frac{mnv^2}{2t}$$
35. If a pump delivers V litres of water over a height of h metres in one minute, then the power of the engine (P) =  $\frac{Vgh}{60}$ .
36. A motor sends a liquid with a velocity 'V' in a tube of cross section 'A' and 'd' is the density of the liquid, then the power of the motor is
- $$P = \frac{1}{2} AdV^3$$
37. The power of the lungs = K.E. of air blown per second.
- $$= \frac{1}{2} (\text{mass of air blown per second}) \times (\text{velocity})^2$$
- $$= \frac{1}{2} \left( \frac{m}{t} \right) v^2$$
38. The power of the heart = pressure  $\times$  volume of blood pumped per second.
39. The capacity to do work is called **energy**. Work and energy have the same units.
40. Potential energy of a body or system is the capacity for doing work, which is possessed by the body or system by virtue of the relative positions of its parts.
41. Water stored in a dam, stretched rubber cord, wounded spring of a clock or toy, etc. possess potential energy.
42. A wounded spring (such as in a clock or toy car) has potential energy  $U = \frac{1}{2} K\theta^2$  (K is a torque constant and  $\theta$  is the number of radians through which it is wound).

**SPRINGS:**

43. Stretched or compressed spring possesses P.E.

a) Elastic potential energy of a stretched spring =  $\frac{1}{2} kx^2 = \frac{1}{2} \frac{F^2}{k} = \frac{1}{2} Fx$



Where  $k = \text{Force constant} = \frac{F}{x}$

(S.I unit of 'K' is  $\text{Nm}^{-1}$ )

b) Work done in increasing the elongation of a spring from  $x_1$  and  $x_2$  is  $\frac{1}{2}k(x_2^2 - x_1^2)$

44. The energy possessed by a body by virtue of its motion is called **kinetic energy**. It is measured by the amount of work which the body can do before coming to rest.
45. Running water, a released arrow, a bullet fired from a gun, blowing wind, etc. possess kinetic energy.
46. If a body of mass  $m$  is moving with a velocity  $v$ , then its kinetic energy  $= \frac{1}{2}mv^2$ .
47. A flying bird possesses both K.E. and P.E.
48. The work done on a body at rest in order that it may acquire a certain velocity is a measure of its kinetic energy.
49. If the kinetic energy of a body of mass  $m$  is  $E$  and its momentum is  $P$ , then  $E = \frac{P^2}{2m}$ .
50. If the momentum of the body increased by 'n' times, K.E increase by  $n^2$  times.
51. If the K.E of the body increases by 'n' times, the momentum increases by  $\sqrt{n}$  times.
52. a) If the momentum of the body increases by  $p\%$ , % increase in K.E.  $= \left(2 + \frac{p}{100}\right) p\%$   
 b) If the momentum of the body decreases by  $p\%$ , % decrease in K.E.  $= \left(2 - \frac{p}{100}\right) p\%$ .
53. a) If the K.E of the body increases by  $e\%$ , % increase in momentum  $= \left(\sqrt{1 + \frac{e}{100}} - 1\right) 100\%$ .  
 b) If the K.E of the body decreases by  $e\%$ , % decrease in momentum  $= \left(1 - \sqrt{1 + \frac{e}{100}}\right) 100\%$ .
54. If two bodies, one heavier and the other lighter are moving with the same momentum, then the lighter body possesses greater kinetic energy.
55. If two bodies, one heavier and the other lighter have the same K.E. then the heavier body possesses greater momentum.
56. Two bodies, one is heavier and the other is lighter are moving with the same momentum. If they are stopped by the same retarding force, then
- the distance travelled by the lighter body is greater. ( $s \propto \frac{1}{m}$ )
  - They will come to rest within the same time interval
57. Two bodies, one is heavier and the other is lighter are moving with same kinetic energy. If they are stopped by the same retarding force, then
- The distance travelled by both the bodies are same.
  - The time taken by the heavier body will be more. ( $t \propto \sqrt{m}$ )
58. Two bodies, one is heavier and the other is lighter are moving with same velocity. If they are stopped by the same retarding force, then.
- The heavier body covers greater distance before coming to rest. ( $s \propto m$ )
  - The heavier body takes more time to come to rest. ( $t \propto m$ )

58. **Simple pendulum :** If the bob (mass  $m$ ) of a pendulum of length ( $l$ ) is raised to a vertical height ( $h$ ) and then released, it executes SHM for smaller angles. The total energy is constant at all positions.

a) At the mean position,  $KE = \frac{1}{2}mv^2$  (max),  $PE=0$ (min)

b) At the extreme position,  $KE = 0$  (min),  
 $PE = mgl(1 - \cos\theta)$  (max)

c)  $KE$  at the mean position =  $PE$  at the extreme position

$$\frac{1}{2}mv^2 = mg(1 - \cos\theta)$$

velocity at equilibrium position,  $v = \sqrt{2gl(1 - \cos\theta)}$ ,

d) When a pendulum of length  $l$  is held horizontal and released.

Velocity at mean position,  $v = \sqrt{2gl}$

e) The graphs for  $PE$  and  $KE$  are parabolic in shape.

#### Rebounding body :

f) If a body falling from height  $h_1$  loses  $x\%$  of energy during the collision with the ground, the height to which it rebounds is

$$h_2 = \left(\frac{100-x}{100}\right)h_1 = \left(1 - \frac{x}{100}\right)h_1$$

g) If a ball strikes a floor from a height  $h_1$  and rebounds to a height  $h_2$ .

$$\% \text{ loss of energy} = \frac{h_1 - h_2}{h_1} \times 100$$

59. **Projectile :**

a) The  $PE$  at maximum height is maximum,  $PE_H = mgH = mg\left(\frac{u^2 \sin^2 \theta}{2g}\right) = \frac{1}{2}mu^2 \sin^2 \theta = E \sin^2 \theta$

b) The  $KE$  at the highest point is minimum.

$$KE_H = \frac{1}{2}m(u \cos\theta)^2 = \frac{1}{2}mu^2 \cos^2 \theta = E \cos^2 \theta$$

c) Total energy =  $PE_H + KE_H =$

$$\frac{1}{2}mu^2 \sin^2 \theta + \frac{1}{2}mu^2 \cos^2 \theta. \Rightarrow E = \frac{1}{2}mu^2$$

d) The ratio of potential and kinetic energies of a projectile at the highest point is  $\tan^2 \theta$ .

$$\frac{P.E_H}{K.E_H} = \tan^2 \theta$$

#### RECOIL OF A GUN:

60. If a bullet of mass ' $m$ ' travelling with a muzzle velocity, is fired from a rifle of mass ' $M$ ', then

i) Velocity of recoil of the gun is  $V = mv/M$

ii) K.E of the bullet is greater than the K.E of the rifle.

$$\text{iii) } \frac{KE_b}{KE_r} = \frac{M}{m} = \frac{v}{V}$$

iv) When a gun of mass ' $M$ ' fire a bullet of mass ' $m$ ' releasing a total energy ' $E$ '.

$$\text{Energy of bullet } E_b = \frac{E.M}{M+m}$$

$$\text{Energy of gun } E_G = \frac{E \cdot m}{M + m}$$

**BALLISTIC PENDULUM:**

61. A block of mass 'M' is suspended by a string and a bullet of mass 'm' is fired into the block with a velocity 'v'. If the bullet embeds in the block, then

i) The common velocity of the system after the impact is  $V = \frac{mv}{M+m}$

ii) The height to which it will rise is  $h = \frac{1}{2g} \left( \frac{mv}{M+m} \right)^2$

62. **Work–energy theorem** : The work done by the resultant force acting on a body is equal to the change in its kinetic energy.

$$W = Fs;$$

$$W = \frac{1}{2}m(v^2 - u^2) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

63. In general, the work done = change in energy.

64. Stopping distance of a vehicle is directly proportional to the square of its velocity and inversely proportional to the braking force.

65. If a body is thrown on a horizontal plane and comes to rest after travelling a distance 's', then

$$\mu m g s = \frac{1}{2} m v^2$$

' $\mu$ ' coefficient of friction

distance travelled before coming to rest

$$s = \frac{\frac{1}{2} m v^2}{\mu m g} = \frac{\text{Initial K.E.}}{\text{retarding force}}$$

66. When a body of mass m falls freely from a height, its total energy is mgh.

When it falls through a distance x, its K.E. is mgx and P.E. is mg(h - x).

67. A stone of mass 'm' falls from a height 'h' and buries deep into sand through a depth 'x' before coming to rest. The average force of resistance offered by sand is  $F = \frac{mg(h+x)}{x} = mg \left( 1 + \frac{h}{x} \right)$ .

68. For a freely falling body or for a body thrown up K.E. at the ground is equal to the P.E. at the maximum height.

69. The total energy of a system is constant. Energy can neither be created nor destroyed. But it can be converted from one form to the other.

Examples on conversion of energy :

1. Electrical → Heat, Eg. Iron, geyser, over

2. Electrical → Light, Eg. Filament bulb,

Fluorescent tube

3. Electrical → Sound, Eg. Loud speaker,

Telephone receiver

4. Electrical → Mechanical. Eg. Fan, Motor

5. Heat → Electrical. Eg : Thermal power plant

6. Heat → Mechanical, Eg. Steam locomotive

7. Mechanical → Electrical. Eg : Dynamo (Generator)

8. Sound → Electrical. Eg: Microphone

9. Light → Electrical. Eg: Photoelectric effect

10. Chemical → Electrical. Eg. Primary cell

70. **Rest mass energy** : Every body or matter possesses a certain inherent amount of energy called rest energy even if it is at rest (so that K.E.= 0) and is not being acted on by a force (so that P.E.= 0). This rest mass energy is given by  $E = mc^2$ .

नास्ति