COLLISIONS

Synopsis:

1. **Collision** is an interaction between two or more bodies in which sudden changes of momenta take place. e.g. : Striking a ball with a bat.
2. Newton's third law of motion leads to the law of conservation of momentum (momentum can neither be created nor destroyed).
3. The momentum of a system $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ remains constant so long as no external forces act on it.
4. Rocket, jet plane, etc. work on the law of conservation of linear momentum.
5. When a bullet of mass m is fired from a gun of mass M with a velocity v, then the gun recoils with a velocity mv/M.
6. When a shot is fired from a gun, the momentum of the shot and the momentum of the gun are equal in magnitude but opposite in direction.
7. K.E. of gun : K.E. of bullet = mass of bullet : mass of gun. The velocity of a bullet is determined by ballistic pendulum.
8. When a shot of mass m with a velocity v gets embedded in a block of mass M free to move on a smooth horizontal surface, then their common velocity = mv/(m+M).
9. A bullet of mass m moving with a velocity v strikes a block of mass M hanging vertically. After striking, the bullet gets embedded in the block and both rise to a height 'h'. Then the velocity of the bullet is given by the formula $v = \left(\frac{m+M}{m}\right)\sqrt{2gh}$.
10. If a boy of mass 'm' walks a distance 's' on a stationary boat of mass 'M', then the boat moves back through a distance of $\frac{ms}{M+m}$.
11. When a moving shell explodes, its total (vector sum) momentum remains constant but its total kinetic energy increases.
12. If the velocities of colliding bodies before and after infact are confined to a straight line, it is called **head on collision** or **one dimensional collision**.
13. **Elastic collisions :**
    1. Both kinetic energy and linear momentum are conserved.
    2. Total energy is constant.
    3. Bodies will not be deformed.
    4. The temperature of the system does not change.

    e.g. Collisions between ivory balls; molecular, atomic and nuclear collisions.
14. **Perfect elastic collisions** :
    1. Two bodies of equal masses suffering one dimensional elastic collision, exchange their velocities after collision. i.e., if $m_1 = m_2$ then $v_1 = u_2$ and $v_2 = u_1$.
    2. If a body suffers an elastic collision with another body of the same mass at rest, the first is stopped dead, whereas the second moves with the velocity of the first. i.e, if $m_1 = m_2$ and $u_2 = 0$ then $v_1 = 0$; $v_2 = u_1$.
    3. When a very light body strikes another very massive one at rest, the velocity of the lighter body is almost reversed and the massive body remains at rest. i.e., if $m_2 >> m_1$ and $u_2 = 0$, then $v_1 = -u_1$ and $v_2 = 0$.
    4. When a massive body strikes a lighter one at rest, the velocity of the massive body remains practically unaffected where as the lighter one begins to move with a velocity nearly double as much as that of the massive one. i.e., if $m_1 >> m_2$ and $u_2 = 0$, then $v_1 = u_1$ and $v_2 = 2u_1$.
    5. When m_1, m_2 are moving with velocities u_1, u_2 and v_1, v_2 before and after collisions, then

a) $v_1 = u_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right) + u_2\left(\frac{2m_2}{m_1 + m_2}\right)$;

b) $v_2 = u_1\left(\frac{2m_1}{m_1 + m_2}\right) + u_2\left(\frac{m_2 - m_1}{m_1 + m_2}\right)$

15. Two bodies of equal masses moving in opposite directions with the same speed collide, if the collision is elastic, each body rebounds with the same speed.
16. A body collides with another body of equal mass at rest. The collision is oblique and perfectly elastic. The two bodies move at right angles after collision.
17. A body of mass m_1 collides head on with another body of mass m_2 at rest. The collision is perfectly elastic. Then

    a) Fraction of kinetic energy lost by the first body is $\frac{4m_1m_2}{(m_1 + m_2)^2}$.

    b) Fraction of kinetic energy retained by first body is $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$.
18. The loss of energy of the first body is maximum 100% when $m_1 = m_2$.
19. **Inelastic collision :**
    1. Linear momentum is conserved.
    2. Kinetic energy is not conserved.
    3. Total energy is conserved.
    4. Temperature changes.
    5. The bodies may be deformed.
    6. The bodies may stick together and move with a common velocity after collision
    7. If the bodies collide and move together after collision; the collision is **perfectly inelastic.**
    8. Two bodies collide in one dimension. The collision is perfectly inelastic, then $m_1u_1 + m_2u_2 = (m_1 + m_2)\,V$
    9. Common velocity after collision $v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)}$
    10. Total loss of kinetic energy in perfect inelastic collision

    $$= \frac{1}{2}.\frac{m_1m_2(u_1 - u_2)^2}{m_1 + m_2} = \frac{1}{2}\frac{m_1m_2}{m_1 + m_2}(u_1 - u_2)^2$$
20. **Ballistic Pendulum:** A block of mass M is suspended by a light string. A bullet of mass m moving horizontally with a velocity 'v' strikes the block and gets embedded in it. The block and the bullet rise to a height h. Then

    a) mv = (M+m)V

    b) $V = \sqrt{2gh}$

    c) $mv = (M+m)\sqrt{2gh}$

    d) $v = \frac{(M+m)}{m}\sqrt{2gh}$
21. If the string of the ballistic pendulum makes an angle θ with vertical after impact and the length of the string is *l* (when θ≤ 90°)

    $$v = \frac{M+m}{m}\sqrt{2gl(1-\cos\theta)}$$
22. If the ballistic pendulum just completes a circle in the plane, velocity of the bullet

    $$v = \frac{M+m}{m}\sqrt{5gl}$$

23. A ballistic pendulum can be used to determine the velocity of projectiles.
24. Fraction of the kinetic energy lost in the impact of a ballistic pendulum is $\frac{M}{(m+M)}$
25. **Coefficient of restitution (e)** : The coefficient of restitution between two bodies in a collision is defined as the ratio of the relative velocity of separation after collision to the relative velocity of their approach before their collision.

   (i) $e = \frac{\text{relative velocity of separation}}{\text{relative velocity of approach}}$

   (ii) $e = -\frac{v_2 - v_1}{u_2 - u_1}$

   (iii) $e = \frac{v_2 - v_1}{u_1 - u_2}$

   Eg : The value of e is 0.94 for two glass balls,
   0.2 for two lead balls.

   (iv) for a perfectly elastic collision, e = 1
   (v) for a perfectly inelastic collision, e = 0
   (vi) If a body falls from a height h_1 on to a hard floor and rebounds to a height h_2, then

   $e = \sqrt{\frac{h_2}{h_1}}$.

26. The value of coefficient of restitution is independent of the masses and the velocities of the colliding bodies. It depends on their materials.
27. If a body dropped from a certain height takes a time t_1 to strike the ground and time t_2 to rise $e = t_2/t_1$.
28. If a body dropped from a certain height hits the ground. With a velocity v_1 and rebounds with a velocity v_2.

   $e = \frac{v_2}{v_1}$

29. A small metal sphere falls freely from a height h upon a fixed horizontal floor. If e is the coefficient of restitution,
   i) the height to which it rebounds after n collisions is $h_n = he^{2n}$
   ii) the total distance travelled by it before it stops rebounding.

   $d = h\left(\frac{1+e^2}{1-e^2}\right)$

   iii) the velocity with which it rebounds from the ground after n^{th} collision is

   $V_n = (\sqrt{2gh})e^n$

   iv) the time taken till it comes to rest is

   $t = \sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$

30. A ball is projected with velocity 'u' at an angle 'θ' to the horizontal plane. It will keep rebounding from the plane for a time $\frac{2u\sin\theta}{g(1-e)}$ (e is the coefficient of restitution)
31. In the above case, its horizontal range before coming to rest is

   $R = \frac{u^2 \sin 2\theta}{g(1-e)}$.

32. **Inelastic collision :**

a) There is loss of energy from the system in the form of heat, sound, light etc.,

b) For the above case $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

c) $\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2v_2^2 < \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$

d) Relative velocity of separation
$v_2 - v_1 = e\,(u_1 - u_2)$

e) $V_1 = u_1\frac{(m_1 - em_2)}{m_1 + m_2} + u_2\frac{(1+e)m_2}{m_1 + m_2}$

$V_2 = u_1\frac{m_1(1+e)}{m_1 + m_2} + u_2\frac{(m_2 - em_1)}{m_1 + m_2}$

V_1 & V_2 are velocities of m_1 and m_2 after the collision

f) Loss of Kinetic energy from the system is

g) $\Delta K = E_{lost} = \frac{1}{2}\left[\frac{m_1m_2}{m_1 + m_2}\right][1 - e^2]\,[u_1 - u_2]^2$