

# FRICTION

## Synopsis :

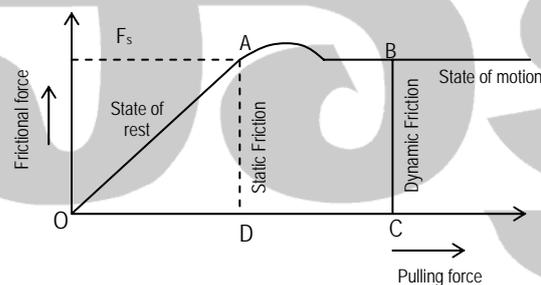
1. When a body is in motion over another surface or when an object moves through a viscous medium like air or water or when a body rolls over another, there is a resistance to the motion because of the interaction of the object with its surroundings. Such a resistance force is called force of **friction**.
2. Friction is a result of molecular interaction. According to modern view, the cause of friction is largely due to atomic and molecular forces between the two surfaces at the point of contact.

## TYPES OF FRICTIONAL FORCE :

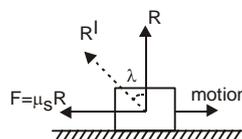
3. **STATIC FRICTION** : The frictional force, which is effective before motion starts between two planes in contact with each other, is known as static friction.

Note:

- 1) Static frictional force is a self adjusting one.
  - 2) The maximum frictional force when the body is ready to start is called **limiting frictional force**.
4. **DYNAMIC FRICTION**: The frictional force, which is effective when two surfaces in contact with each other are in relative motion with respect to each other, is known as dynamic friction.
  5. **ROLLING FRICTION**: The frictional force, which is effective when a body rolls or rotates on a surface, is known as rolling friction.



6. **Limiting friction** ( $F_s$ ) is independent of the area of contact of the surfaces. Rolling friction depends on the area of contact.
7. **Limiting friction** is directly proportional to the normal reaction between the surfaces in contact.



$$F_s \propto R$$

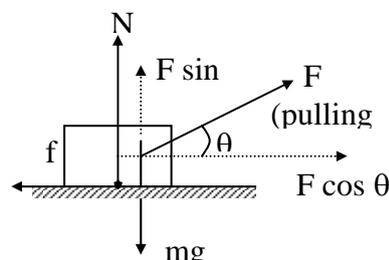
$F_s = \mu_s R$  where  $\mu_s$  is called the **coefficient of static friction**. It depends upon the nature of the surfaces in contact and their state of roughness.

8. The angle between  $R$  and the resultant of  $R$  and  $F$  (i.e.,  $R'$ ) is called the **angle of friction**  $\lambda$ .

$$\tan \lambda = \mu$$

9. **Characteristics of static friction:**

- a)  $\mu_s$  between two given surfaces is independent of the normal force between the two surfaces.
- b)  $\mu_s > 0$ , it can also be greater than one, but in most of the cases it is less than one
- c) If  $\theta_s$  is the angle of limiting friction between two surfaces  $\tan \theta_s = \mu_s$
10. When one body moves over the other, the force of friction acting between the two surfaces is called **kinetic friction**.
11. The force of kinetic friction is independent of the area of the surfaces in contact and is proportional to the normal reaction  $F_k \propto R$ .  
 $F_k = \mu_k \cdot R$   
 Where  $\mu_k$  is coefficient of kinetic friction.
12. When one body moves over another body, the coefficient of friction is less than limiting coefficient of friction and is called the **coefficient of kinetic friction**.
13.  $F_k$  is independent of the velocity of sliding provided the velocity is low.
14. When a body rolls over another, the frictional force developed is called rolling frictional force and the corresponding coefficient of friction is called **coefficient of rolling friction** ( $\mu_r$ ).
15. **Rolling friction:**
- a) Rolling friction comes into play when a body such as a wheel rolls on a surface.
- b) Rolling friction arises out of the deformation of the two surfaces in contact with each other.
- c) Greater the deformation greater is the rolling frictional force.
- d) The rolling frictional force is inversely proportional to the radius of the rolling body.
- e) The rolling frictional force between two given surfaces is lesser than kinetic and limiting frictional forces.
- f) If  $\mu_R$  is the coefficient of rolling friction  
 $\mu_R < \mu_k < \mu_s$  for a given pair of surfaces.
- g) Ball bearings are used in machinery parts because rolling friction is least.
- h) Radial tyres used in cars reduce rolling friction.
16. When lubricants or viscous liquids are introduced between the surfaces of two solids in contact, they reduce frictional forces because intermolecular forces in liquids are much weaker than those in solids.
17. **Pulling a block or roller**
- a) If the pulling force is such that  $F \cos \theta < f_s$ , ( $f_s$  is limiting friction) the block will be at rest and the force of friction between block and the surface is  $f = F \cos \theta$



- b) The normal force is  $N = mg - F \sin \theta$
- c) Force needed to just slide the body is

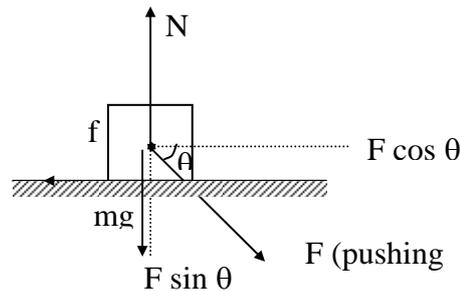
$$F = \frac{\mu_s mg}{\cos \theta + \mu_s \sin \theta} = \frac{mg \sin \phi}{\cos(\theta - \phi)}$$

Where  $\phi$  is the angle of friction between the two surfaces.

- d) If the applied force is greater than the above value, block slides with acceleration and the force of friction between the block and the surface is  $f_k$ .
- e) The minimum possible force among all directions required to just move the body is  $mg \sin \phi$  (or)  $\frac{mg\mu_s}{\sqrt{1+\mu_s^2}}$  where  $\phi$  is the angle of friction. The force must be applied at angle  $\theta$  to the horizontal at an angle equal to angle of friction  $\phi$ .

### 18. Pushing a block or Roller:

- a) If the pushing force is such that,  $F \cos \theta < f_s$ , the block will be rest and the force of friction between the block and the surface is  
 $f = F \cos \theta$ .



- b) The normal force is  $N = mg + F \sin \theta$   
 c) Force needed to just slid the body is

$$F = \frac{\mu_s mg}{\cos \theta - \mu_s \sin \theta} = \frac{\mu_s mg \sin \phi}{\cos(\theta + \phi)}$$

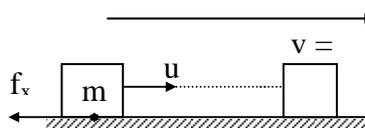
where  $\phi$  is the angle of friction.

- d) Pulling is easier than pushing because lower frictional force, in the case of pulling need to be overcome.
- e) If the angle made by the pushing force with the vertical is lesser than or equal to angle of friction, the block cannot be moved, irrespective of the magnitude of the applied force.
19. A uniform chain of length  $L$  lies on a table. If the coefficient of friction is  $\mu$ , then the maximum length of the chain which can overhang from the edge of the table without sliding down is  $\frac{\mu L}{\mu + 1}$ .

### 20. Block on a rough fixed horizontal surface

- a) If we continue to apply a force  $F = f_s$ , the block slides with an acceleration given by  
 $a = (\mu_s - \mu_k) g$
- b) Once the block slides, force of friction on the block is kinetic frictional force ( $f_k$ )
- c) If the block slides with an acceleration under the influence of an external force  $F$ , the acceleration of the block is  $a = \frac{F - f_k}{m}$

### 21. Sliding block on a horizontal surface coming to rest:



- a) If a block having initial velocity  $u$  slides on a rough horizontal surface and comes to rest, the acceleration of the block is  $a = -\mu_k g$
- b) Distance traveled by the block before coming to rest is  $S = \frac{u^2}{2\mu_k g}$

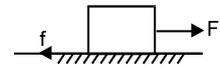
c) Time taken by the block to come to rest is

$$t = \frac{u}{\mu_k g}$$

## 22. Motion on a rough horizontal plane :

(a) Pulled with a horizontal force F:

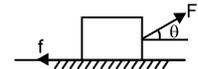
(i) body moving with uniform velocity  $F = \mu_k mg$ .



(ii) body moving with uniform acceleration  $F = m(\mu_k g + a)$ .

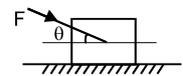
(b) Pulled with a force F inclined at an angle  $\theta$  with the horizontal and the body moving with uniform velocity.

$$F = \frac{\mu_k mg}{\cos \theta + \mu_k \sin \theta}$$



c) Pushed with a force F inclined at an angle  $\theta$  with the horizontal and the body moving with uniform velocity:

$$F = \frac{\mu_k mg}{\cos \theta - \mu_k \sin \theta}$$



## 23. Block on a rough inclined plane

a) **Angle of repose  $\alpha$ :** It is the angle of inclination of the inclined plane with the horizontal for which block just begins to slide down.

b) If  $\alpha$  is the angle of repose  $\mu_s = \tan \alpha$

c) The angle of repose is the angle of static friction

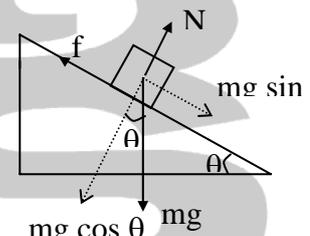
d) The angle of inclination is ( $\theta$ ) less than ( $\alpha$ ), the block does not slide down, it is at rest. The force of friction  $f < f_s$  and is equal to

$$f = mg \sin \theta \quad [mg \sin \theta < f_s]$$

e) If the angle of inclination is [ $\theta$ ] equal to [ $\alpha$ ]. Then the block is in limiting equilibrium. The force of friction is

$$f = f_s = \mu_s mg \cos \alpha \quad [mg \sin \theta = f_s]$$

f) If the block slides down the inclined plane with uniform velocity  $\mu_k = \tan \theta$  where  $\theta$  is the angle of inclination of the inclined plane.



### Sliding down the inclined plane:

g) If the inclination is maintained at  $\alpha$ , the block will eventually slide down with an acceleration equal to  $a = g \frac{[\mu_s - \mu_k]}{\sqrt{1 + \mu_s^2}}$

h) If  $\theta \geq \alpha$ , the block slides down with an acceleration given by

$$a = g [\sin \theta - \mu_k \cos \theta] \quad [mg \sin \theta > f_s]$$

i) If  $\theta \geq \alpha$ , and the block slides down from the top of the inclined plane. Velocity at the bottom of the plane is

$$V = \sqrt{2gl(\sin \theta - \mu_k \cos \theta)} = \sqrt{2gh(1 - \mu_k \cot \theta)}$$

j) In the above case time of descent is

$$t = \sqrt{\frac{2L}{g(\sin \theta - \mu_k \cos \theta)}}$$

- k) The time taken by a body to slide down on a rough inclined plane is 'n' times the time taken by it to slide down on a smooth inclined plane of same inclination and length, then coefficient of friction is  $\mu = \tan \theta \left[ 1 - \frac{1}{n^2} \right]$

### Moving up the inclined plane:

- l) If a block is projected up a rough inclined plane, the acceleration of the block is  
 $a = -g [\sin \theta + \mu_k \cos \theta]$
- m) Force opposing the motion of the block is  
 $F = mg \sin \theta + \mu_k mg \cos \theta$
- n) The distance traveled by the block up the plane before the velocity becomes zero is  

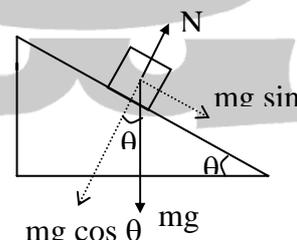
$$S = \frac{u^2}{2g(\sin \theta + \mu_k \cos \theta)}$$
- o) The time of ascent is  $t = \frac{u}{g(\sin \theta + \mu_k \cos \theta)}$ . In the above case the block will come down sliding only if  $\theta \geq \alpha$ .
- p) In the above case if time of descent is n times the time of ascent, then  

$$\mu = \tan \theta \left[ \frac{n^2 - 1}{n^2 + 1} \right]$$
- q) Force needed to be applied parallel to the plane to move the block up with constant velocity is  $F = mg \sin \theta + \mu_k mg \cos \theta$
- r) Force needed to be applied parallel to the plane to move the block up with an acceleration a is  
 $F = mg \sin \theta + \mu_k mg \cos \theta + ma$
- s) If block has a tendency to slide, the force to be applied on the block parallel and up the plane to prevent the block from sliding is  $F = mg \sin \theta - \mu_s mg \cos \theta$

### 24. Block on a smooth inclined plane

- a)  $N = mg \cos \theta$
- b) Acceleration of sliding block  
 $(a = g \sin \theta)$
- c) If  $l$  is the length of the inclined plane and  $h$  is the height. The time taken to slide down starting from rest from the top is

$$t = \sqrt{\frac{2l}{g \sin \theta}} = \frac{1}{\sin \theta} \sqrt{\frac{2lh}{g}}$$



- d) Sliding block takes more time to reach the bottom than to fall freely from the top of the incline.
- e) Velocity of the block at the bottom of the inclined plane is  
 $V = \sqrt{2gl \sin \theta} = \sqrt{2gh}$  same as the speed attained if block falls freely from the top of the inclined plane.
- f) If a block is projected up the plane with a velocity  $u$ , the acceleration of the block is  
 $a = -g \sin \theta$
- g) Distance traveled up the plane before its velocity becomes zero is

$$S = \frac{u^2}{2g \sin \theta}$$

- h) Time of ascent is  $t = \frac{u}{g \sin \theta}$

25. When the body moves on a rough horizontal surface, the force of friction is  $\mu mg$ . If  $s$  is the displacement,  
the work done against friction =  $\mu mgs$ .  
This work is converted into heat.

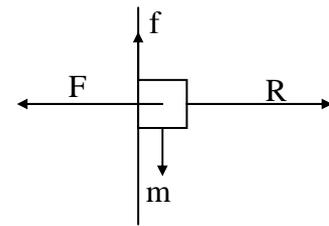
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**26. Block pressed against a vertical wall :**

A body of mass 'm' is pressed against a vertical wall with a horizontal force 'F'.  
The normal force is F.

If the coefficient of static friction is  $\mu_s$ , then

- Block will be about to slide down if  $\mu_s F = mg$ .
- If  $\mu_s F \geq mg$ , block will not slide and the frictional force acting on the block is  $mg$ .
- If  $\mu_s F < mg$ , block will slide and the frictional force acting on the block will be  $\mu_s F$ .



27. A vehicle is moving on a horizontal surface. A block of mass 'm' is stuck on the front part of the vehicle. The coefficient of friction between the truck and the block is ' $\mu$ '. The minimum acceleration with which the truck should travel, so that the body may not slide down is  $a = g/\mu$ .

**28. Block in a lorry:**

- When a block is lying on the floor of an accelerating lorry, the force of friction acting on the block is in the direction of acceleration of the lorry.
- Relative to lorry, block experiences a pseudo force  $ma$  opposite to the acceleration of the lorry ( $a =$  acceleration of lorry)
- The maximum acceleration of the lorry for which block begins to slide on the floor of the lorry is

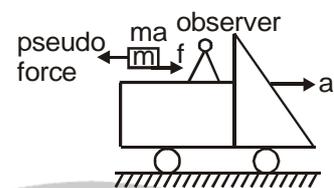
$$a = \mu_s g$$

$$[ma = \mu_s mg \therefore a = \mu_s g]$$

- If  $a < \mu_s g$  block does not slide and friction force on the block is  $f = ma$
- If  $a \geq \mu_s g$  block slips or slides on the floor. The acceleration ( $a'$ ) of the block relative to lorry is  $a' = a - \mu_k g$

$$\begin{cases} ma - fk = ma' \\ ma - \mu_k mg = ma' \\ \therefore a' = a - \mu_k g \end{cases}$$

- In the above case, acceleration of the block relative to earth is  $\mu_k g$ .

**Block on Block:**

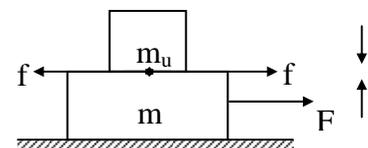
29. **Case I:** (lower block pulled and there is no friction between lower block and the horizontal surface)

- When the lower block is pulled upper block is accelerated by the force of friction acting upon it
- The maximum acceleration of the system of two blocks for them to move together without slipping is  $a = \mu_s g$ , where  $\mu_s$  is the coefficient of static friction between the two blocks.
- If  $a < \mu_s g$  blocks move together and applied force is  $F = (m_B + m_u) a$
- If  $a < \mu_s g$  frictional force between the two blocks  $f = m_u a$
- The maximum applied force for which both blocks move together is

$$F_{\max} = \mu_s g (m_u + m_B)$$

- If  $F > F_{\max}$  blocks slip relative to each other and have different accelerations. The acceleration of the upper block is  $\mu_k g$  and lower block is

$$a = \frac{F}{m_B + m_u}$$



30. **Case - II (Upper block pulled and there is no friction between lower block and the horizontal surface)**

a) When the upper block is pulled, lower block is accelerated by the force of friction acting upon it.

b) The maximum acceleration of the system of two blocks for them to move together without slipping is  $a_{\max} =$

$$\mu_s \frac{m_u}{m_B} g \quad (\mu_s = \text{coefficient of static friction between the two blocks})$$

c) If  $a < a_{\max}$  frictional force between the two blocks is  $f = M_B a$

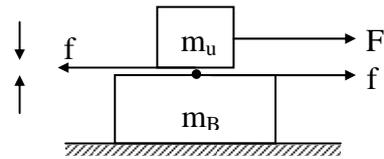
d) If  $a < a_{\max}$  then applied force on the upper block is  $F = (m_B + m_u) a$

e) The maximum force for which both blocks move together is  $F_{\max} = \mu_s \frac{m_u}{m_B} g (m_u + m_B)$

f) If  $F > F_{\max}$  blocks slide relative to each other and hence have different accelerations.

The acceleration of the lower block is  $\mu_k \frac{m_u}{m_B} g$  and the acceleration of the upper block

$$\text{is } \frac{(F - \mu_k m_u g)}{m_u}.$$



31. When moisture is present between bodies, friction increases.

32. If the metal surfaces of ball bearing are not hard, friction will be high.

33. friction is reduced by using alloys for making the moving parts (alloys have low coefficient of friction)

34. **Advantages of friction:**

i) Safe walking on the floor is possible because of the friction between the floor and the feet.

ii) Nails and screws are driven in the walls or wooden surfaces due to friction.

iii) Friction help the fingers to hold a drinking water tumbler or pen.

iv) Vehicles move on the roads without sliding due to friction and they can be stopped due to friction.

v) The mechanical power transmission of belt drive is possible due to friction.

35. **Disadvantages of friction:**

i) Friction results in the large amount of power loss in engines.

ii) Due to friction, the wear and tear of the machine increases.

iii) Due to friction, heat is generated which goes as a waste.

36. **Methods of reducing friction:**

i) Friction between two surfaces of contact can be reduced by polishing the surfaces.

ii) A lubricant is a substance which forms a thin layer between two surfaces in contact and reduces the friction. The process of reducing friction is called lubrication. Soap water, two-in-one oil and grease are the examples of lubricants.

iii) The free wheels of vehicles like cycles, two wheelers, motor cars, shafts of motors, dynamos etc., are provided with ball bearing to reduce the friction.

iv) Automobiles and aeroplanes have special construction i.e. they are stream lined to reduce the friction due to air.