GRAVITATION

Synopsis :

1. All forces in nature can be classified under three categories depending upon their relative strengths. They are
   (i) gravitational force,
   (ii) electromagnetic force and
   (iii) nuclear force.
2. **Fundamental forces of the universe:**
   a) **Gravitational Force:**
      i) It is the weakest of all the forces but has the longest range.
      ii) It is because of attraction between particles due to the property of mass.
      iii) Since it is a weak force, the force effects are considerable only when the interacting objects are massive.
      iv) It provides the large scale structure for the universe.
   b) **Electromagnetic force:**
      i) It is a strong force between two charged particles and has a long range.
      ii) It acts through electric and magnetic fields.
      iii) It can be attractive as well as repulsive.
      iv) According to quantum field theory electromagnetic force between two charges is mediated by exchange of **Photons.**
   c) **Nuclear force:**
      i) They are a short range, strong force of attraction between nucleons, which provides stability to the nucleons.
      ii) It is the strongest of all the fundamental forces and has a range of 1 fermi = 10^{-15} m.
3. a) **Order of Range**
   Range of Gravitational force > Range of Electromagnetic force > Range of nuclear force.
   b) **Order of strength:**
   Nuclear force > Electromagnetic force > Gravitational force
4. The ratio of relative strengths of nuclear, electromagnetic and gravitational forces is 1 : 10^{-15} : 10^{-35}.
5. Aryabhat in his famous book "Aryabhatiyam" suggested that earth is a solid sphere and it spins around itself.
6. In Rigveda, paths of planets in solar system were suggested to be elliptical.
7. In "Thaithireeya Aruna Patham", existence of several solar systems moving under the influence of a great central force was suggested.
8. **The geo–centric theory** was proposed by **Ptolemy**. According to this theory, all the planets revolve round the earth in circular orbits, the earth being at the centre.
9. **The helio–centric theory** was proposed by **Copernicus**. According to this theory, all the planets revolve round the Sun in circular orbits, the Sun being at the centre.
10. Kepler's laws confirm the helio–centric theory.

11. **Kepler's first law of motion (Law of orbits) :** All the planets revolve round the Sun in elliptical orbits with the Sun at one of the foci.
12. Planets are nine in number which revolve round the Sun and have self rotation. The order of the planets revolving round the Sun as we move away from the Sun is Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune and Pluto.
13. Jupiter is the biggest planet and Mercury is the smallest planet.
14. The planet which is nearest to the Earth is Venus.
15. The moon is the satellite of the earth. Acceleration due to gravity on the surface of the moon is 1.6 ms^{-2} or 1/6 that of the earth.
16. Mercury and Venus have no satellites, Earth and Pluto have each one satellite, Mars, Jupiter, Saturn, Uranus and Neptune have 2, 16, 22, 12 and 6 satellites respectively.
17. **Kepler's second law :** (**Law of areas**) : The radius vector joining a planet to the Sun sweeps out equal areas in equal intervals of time. ($I\omega$ = constant). This law is a direct consequence of the law of conservation of angular momentum.
    a) A planet moves fastest when it is nearest to the Sun (perihelion or perigee) and moves slowest when it is farthest from the sun (aphelion or apogee).
    b) The line joining the sun and the earth sweeps out equal areas is equal intervals of time i.e. a real velocity is constant.
    c) A real velocity is $\frac{dA}{dt} = \frac{1}{2} r^2 \omega$
    $\frac{dA}{dt} = \frac{L}{2m}$ L is the angular momentum of the planet of mass m in the given orbit.
    d) Kepler's second law is a consequence of law of conservation of angular momentum
    e) According to second law a planet moves faster when it is nearer to sun and moves slower when it is far a way from the sun.
    f) According to II law
    $V_{max} r_{min} = V_{min} r_{max}$

    <u>III law: Law of Periods:</u>

    f) Square of the period of any planet (T^2) about the sun is proportional to cube of the mean distance (R^3) of the planet from the sun.
    $T^2 \alpha R^3$ or T^2/R^3 = constant. $\frac{T_1^2}{R_1^3} = \frac{T_2^2}{R_2^3}$
    g) According to third law, as the distance of the planet increases, duration of the year of the planet increases.
    h) Kepler's laws supported heliocentric or Copernicus theory.
18. The period of revolution of the moon round the earth is equal to the period of its self rotation.
19. **Newton's law of universal gravitation :** Every two bodies in the universe attract each other with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
20. If m_1 and m_2 are the masses of two bodies and d is the distance between them, the gravitational force of attraction F which each exerts on the other is given by

$F = \dfrac{G \cdot m_1 m_2}{d^2}$ where G is called universal gravitational constant and is equal to 6.67×10^{-11} Nm^2kg^{-2}.

21. G was first accurately determined by Cavendish. It is a scalar quantity.
22. **Properties of gravitational force:**
    a) The gravitational force of attraction between two particles from an action and reaction pair, ie equal in magnitude and opposite in direction.
    b) Gravitational force is a central force i.e. it acts along the line joining the two particles.
    c) Gravitational force between two particles is independent of the properties of intervening medium.
    d) Gravitational force between two particles is independent of the presence of other particles.
    e) **Principle of superposition:** If a no. of particles interact with each other, the net force acting on a given particle is the vector sum of the forces acting upon it, due to its interaction with each of the other particles.
    f) They are long range attractive forces.
23. If two identical spheres each of radius r are kept in contact with each other, the gravitational force F between them is proportional to r^4.
24. If two identical spheres each of radius r are separated by a certain distance and the distance between the spheres is maintained constant, the gravitational force F between them is proportional to r^6.
25. Newton's third law of motion do not apply when (i) velocities of moving bodies are comparable to velocity of light and (ii) gravitational fields are very strong, e.g. gravitational field between objects whose masses are greater than the mass of sun.
26. Universal law of gravitation cannot explain the reason for gravity between objects and force of attraction between two bodies even when they are not in physical contact.
27. The relation between g and G is given by $g = \dfrac{GM}{R^2} = \dfrac{4}{3}\pi R\rho G$ where M is the mass of the planet, R is its radius and ρ is the mean density of the planet.
28. The value of g near the equator is 9.78 ms^{-2} and near the poles it is equal to 9.83 ms^{-2} and is zero at the centre of the earth.
29. Variations of g are due to i) shape of the earth (Pear shaped, more flattened at the S–pole than at the N–pole), ii) Spin of the earth, iii) Latitude,
    iv) Altitude and v) Local conditions.
30. **Shape :** Earth is flat at the poles and some what bulky at the equator. The polar radius is lesser than the equatorial radius by 21 km. Hence g is greater at the polar regions than at the equatorial region.
31. **Latitude :**
    i) Because of the spin of the earth, more centrifugal force acts on bodies near the equator. Hence g value is less at the equator.
    ii) Variation of g due to rotation of the earth is given by $g_1 = g - R\omega^2\cos^2\lambda$ where λ = latitude angle, ω = angular velocity of earth.
    iii) The angular velocity of rotation of the earth is 7.27×10^{-5} $rads^{-1}$. The linear velocity of a body at the equator is 0.465 kms^{-1}.
    iv) Spin of the earth does not affect the value of g at the poles.
    v) If the earth stops spinning, g increases slightly near the equator.
    vi) If the earth shrinks without change in its mass, g increases.

vii) The reduction in value of 'g' at the equator is 0.034 ms^{-2} due to the rotation of earth. ($\because R\omega^2 = 0.034$)

viii) If the earth spins at 17 times the present speed, g becomes zero at the equator.

ix) Isograms are the lines joining the places of equal g on the earth.

x) With the help of isograms, mineral deposits and mineral oils are located.

xi) Etova balance, Gradiometer, Gravimeter are the instruments which are used to measure even the slightest variations in g.

32. **Altitude :**

i) As the height from the surface of the earth increases, the value of g decreases.

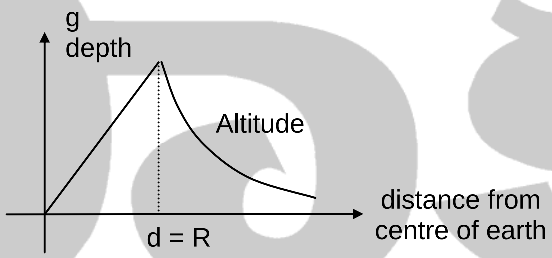
ii) If g is the acceleration due to gravity on the surface of earth and g_h at a height h above the earth, then $g_h = g(1 - \frac{2h}{R})$ approximately or $g_h = \frac{gR^2}{(R+h)^2}$ exactly.

33. **Depth :**

i) As the depth from the surface of the earth increases, the value of g decreases.

ii) If d is the depth below the surface, then $g_d = g(1 - \frac{d}{R})$.

34. Graph of variation of g with distance from the centre of earth:



35. g above the earth's surface is inversely proportional to the square of the distance from the centre of the earth.

36. The value of g at the centre of the earth is zero.

37. The necessary centripetal force for a satellite is provided by the gravitational attraction of the earth.

38. According to **'field concept'** a mass particle modifies the 'space' around it in someway and sets up a gravitational field.

51. **Gravitational field strength** is the force experienced by unit mass in the gravitational field. It is a vector. Its magnitude is equal to the value of g.

$$I = \frac{GM}{R^2}$$

52. **Propagation of gravitational fields :**

a) According to Einstein, gravity is because of distortion of space time due to the presence of matter i.e. Space time is curved because of the presence of matter

b) According to General theory of relativity whenever mass particles are accelerated, the gravitational field around them changes and they are said to produce gravitational waves, which are ripples in Space time.

c) It is difficult to detect gravitational waves, but there are observational consequences such as near a pulsar, Black hole or when a massive star undergoes a gravitational collapse.

d) According to quantum theory all fields are quantum in nature including gravity. According to quantum theory of gravity, gravitational force between two mass particles is mediated by a particle called graviton.

e) A graviton has zero rest mass, travels with the velocity of light, therefore gravitational field propagates with the velocity of light.

53. **Evolution of stars, cold stars (white dwarf Neutron stars) and Black holes:**

Life cycle of a star:

a) A star is formed when, a large amount of interstellar gas, mostly H_2 and He, starts to collapse on itself due to the gravitational attraction between the gas atoms or molecules.

b) As the gas contracts it heats up due to atomic collisions.

c) As the gas continues to contract, the collision rate increases to such an extent, that the gas becomes very hot, and the gas atoms are stripped off their electrons, and the matter is in a completely ionized state, containing bare nuclei and electrons. Such a state of matter is called **plasma state**.

d) Under the conditions specified above, the bare nuclei have enough energy to fuse with each other (Nuclear fusion). Hydrogen nuclei fuse in such a manner to form Helium with a release of a large amount of energy in the form of radiation.

e) The radiation emitted in this process is mostly emitted in the form of visible light, UV light, IR light etc., from its outer surface. This radiation is what causes the star to shine, which makes them visible (Ex. Sun and other visible stars)

f) The star at this stage is halted from gravitational collapse (contraction) since the gravitational attraction of matter in it, towards the centre of the star is balanced by the out ward radiation pressure. A star will remain stable like this for millions of years, until it runs out of nuclear fuel such as H_2 and He.

g) The more massive a star is, faster will be the rate at which it will use its fuel because greater energy is required to balance the greater gravitational attraction, owing to greater mass i.e. massive stars burn out quickly.

h) When the nuclear fuel is over, i.e. when the star cools off, the radiation pressure is not sufficient to halt the gravitational collapse. The star then begins to shrink with tremendous increase in the density. The star eventually settles into a white dwarf, Neutron star or a Black hole depending upon its initial mass.

54. **Formation of white dwarfs:**

a) For a star to become a white dwarf, initial mass must be less than ten solar masses. ($M < 10\ M_s$ where Ms is the mass of the sun)

b) As the star collapses after cooling off, the radiation emitted due to fusion of remaining H_2 nuclei at the outer edge of the core, causes the lighter outer mantle of the star to expand to several times its original diameter. Such an expanding star is called a **"Red Giant"**

c) After several millions of years., the H_2 fuel is exhausted and the material from the outer mantle of the Red giant is down off and the remaining core left over, which is very dim is called "**white dwarf"**.

d) A white dwarf is barely visible and has a mass of less than 1.4 M_s.

e) **Chandrashekar limit:** The maximum mass that a white dwarf can have is called Chandrashekar limit, which is 1.4 Ms. A white dwarf cannot have a mass greater than 1.4 Ms.

f) The volume of a white dwarf is about 10^{-6} times the volume of the original star.

g) In the white dwarf stage further gravitational collapse is halted due to the balance between repulsion of electrons and gravitational attraction. The repulsion between electrons is called **degenerate electron pressure or degeneracy pressure.**

h) The degeneracy pressure is because all the lower available quantum energy states, is filled up by electrons. The pauli exclusion principle prevents further filling up of these energy states. This causes the remaining electrons to fill up higher energy levels causing the required effect. In the white dwarf there is complete break down of atomic structures.

i) Matter in a white dwarf has a very high density. A white dwarf having the mass of the sun has approximately one sixth of the earth.

j) If the mass is greater than 1.4 Ms, the degenerate electron pressure between electrons will not be able to halt further gravitational collapse. The star than collapses into a neutron star or a black hole.

55. **Formation of Neutron Stars:**

a) For a star to become a neutron star, its initial mass must be greater than ten solar masses. ($M>10M_s$)

b) As a star with initial mass > $10M_s$ cools off the large mass of the star causes it to contract abruptly and the temperature of the core rises over 100 billion degrees and when out of fuel it explodes violently. The explosion flings most of the star matter into space and is called a **supernova.** A supernova explosion is very bright and outshines the entire light from the galaxy.

c) The mass of the matter left behind is greater than 1.4 M_s.

d) If the mass of the left over matter is between 1.4 M_s and 3 M_s Neutron stars evolve.

e) When the mass of the left over matter lies in the range 1.4 M_s and 3 M_s, the repulsion between electrons will not be sufficient to stop gravitational collapse. Under such conditions, the protons and electrons present in the star combine to form neutrons. After the formation of neutrons, the outward degeneracy pressure between neutrons prevents further gravitational collapse, and the matter left over is the **Neutron star.**

f) Neutron star, has a definitely much larger than a white dwarf and has a radius of about 20 kms.

g) Neutron stars are also called **pulsars**, because they emit regular pulses of radio waves.

h) Neutron stars are not visible.

56. **Formation of black holes:**

a) Black holes are objects in space, whose gravitational field is so strong that even light cannot escape from it. We cannot see black holes because light emitted by them, would not reach us, however its gravitational effect, will be felt by other objects.

b) The possible existence of a black hole was first pointed out by John Michell (1783).

c) The phrase black hole was first coined by John wheeler.

d) Karl Schwarzschild predicted the existence of a dense object into which other objects could fall, but out of which no object or even light could ever come out. He predicted a magic sphere around this dense object through which nothing can move outwards. The radius of this sphere is known as **Schwarzschild radius (R_s)**.

Schwarzchild radius is $R = \frac{2GM}{c^2}$

M = mass of object G =universal gravitational constant and c is the velocity of light.

e) Any object would be a black hole if and only all of its mass is inside a sphere with a radius equal to Schwarzchild radius. At the Schwarzchild radius escape velocity is equal to the speed of light. This boundary is called the **event horizon.**

f) Any event that occurs, within the event horizon cannot be observed from outside it.

g) Black holes have infinitely large density. The matter present inside is called **singularity.**

h) Any object, even light, present within the event horizon will be sucked into the black hole

i) Stars turn into black holes, when the mass of the remaining matter after a supernova explosion is greater than $3M_s$. The initial mass being greater than $10M_s$.

j) When the mass of the remaining star is greater than $3M_s$ even the degeneracy pressure between neutrons cannot prevent, the gravitational collapse and Black holes are formed.

k) **Evidences for existance of black holes**

i) Cygnus XI is a binary star, which contains a visible star moving around an unseen companion and which emits X rays. It is they believed that the unseen companion is a black hole, which sucks off matter from the visible star. As the matter moves towards the Black hole it gets very hot (about 100 billion degrees) and emits X - rays.

ii) **Quasars** (Quasi Stellar radio sources): Are very distant objects which emit powerful radio waves. A Quasar is an entire galaxy under going a gravitational collapse, due to the presence of a super massive black hole at the galactic center.

57. **Formation of black holes:**

Massive Star
↓ (energy source (hydrogen) decreases)
Red giant stage
↓ (material particles blown off)
White dwarf stage
↓ (Chandrasekhar limit exceeded)
Super nova
↓ (further core collapse)
Neutron star
↓ (further core collapse)
Black hole

58. **Gravitational potential** is the work done in moving a unit mass from infinity to the point under consideration. $V = -\frac{GM}{R}$ or $I = -\frac{dV}{dR}$

59. The binding energy of a mass 'm' at rest on the surface of earth of mass M and radius R is given by $U = -\frac{GMm}{R} = -mgR$.

Generally it is stated as positive energy i.e., as mgR.

60. Gravitational potential due to a mass M at a distance r is $-\frac{GM}{r}$.

61. **Inertial frames of reference :**
    a) Frames of reference in which Newton Laws of motion are applicable.
    b) Inertial frames of reference move with uniform velocity relative to each other.
    c) Force acting on an object due to its interaction with another object is called a real force
    d) All fundamental forces of nature are real.
    e) Real forces form action, reaction pairs.
    Ex: Normal force, Tension, weight, spring force, muscular force etc.
    f) Equation of motion relative to an observer in an inertial frame is $\Sigma\vec{F}_{real} = m\vec{a}$
    (m is the mass of the body having acceleration $\vec{a}$ relative to the observer.
    g) Observers in all inertial frames, measure the same acceleration for a given object but might measure different velocities
    h) Observers in all inertial frames, measure the same net force acting on a given object.
    i) Basic laws of physics are identical in all inertial frames of reference.
    j) Inertial frames of reference are called **Newtonian or Galilean frames of reference**

62. **Non - Inertial frames:**
    a) Frames of reference in which Newton Laws are not applicable.
    b) **Pseudo Force:** Force acting on an object relative to an observer in a non - inertial frame, without any interaction with any other object of the universe.
    c) Pseudo force exist for observers only in non - inertial frames, such forces have no existence relative to an inertial frame.
    d) If $\vec{a}$ is the acceleration of a non - inertial frame. The Pseudo force acting on an object of mass m, relative to an observer in the given non- inertial frame is
    $\vec{F}_{pseudo} = -m\vec{a}$
    i.e. Pseudo force acts on an object opposite to the direction of acceleration of the non - inertia frame.
    e) **Centrifugal force:** It is a pseudo force experienced radically outward by an object relative to the object, moving in a circular path relative to an inertial frame. The centrifugal force is given by $\frac{mv^2}{r}$.
    (V = speed of object relative to inertial frame)

63. **Inertial mass and gravitational mass:**
    a) **Inertial mass (m_1):** The inertial mass of a body is the ratio of the force acting on the body to the acceleration produced by the force.
    b) It is difficult to measure inertial mass.
    c) **Gravitational Mass (m_g) :** If is the ratio of the gravitational force acting on a body to the acceleration due to gravity.
    d) Gravitational mass can be measured using spring balance and common balance
    e) Inertial and gravitational mass of a body are equal.

64. **PRINCIPLE OF EQUIVALENCE**:
When experiments are conducted in inertial and non inertial frames under the same conditions, give the same results, the frames are to be identical. This is the principle of equivalence.

65. **Orbital Velocity :**

The velocity required for a satellite to orbit round the earth very close to it is called **orbital velocity** (v_o) or **first cosmic velocity**.

$V_o = \sqrt{gR} = \sqrt{\frac{GM}{R}}$ = 7.9 kms^{-1} for earth bound satellites.

V_o = 1.7 kms^{-1} for moon bound satellites.

a) The velocity with which a satellite must be projected parallel to the earth surface, after parking it in the given orbit, so that it moves around the earth in a circular orbit.

b) If the orbit radius r, orbital velocity is $V_0 = \sqrt{\frac{GM}{r}}$

c) Orbital velocity is independent of the mass of the satellite.

d) V_0 depends on mass of the planet and radius of the orbit.

e) For an orbit close to earth surface orbital velocity is $V_0 = \sqrt{\frac{GM}{r}} = \sqrt{gR}$

f) V_0 close to earth is 7.92 kms^{-1}.

g) As height increases, orbital velocity decreases

h) The period of a satellite in a circular orbit close to the surface of the earth

$T = 2\pi\sqrt{\frac{R}{g}}$ = 84.6 min

i) Launching of a Rocket is near equator in west to east direction.

66. A satellite of mass m orbiting close to the earth has kinetic energy and potential energy.

67. Kinetic energy of the satellite = $\frac{GMm}{2R} = \frac{mgR}{2}$

68. Potential energy of the satellite = $-\frac{GMm}{R}$

69. Total energy = K.E + P.E = $-\frac{GMm}{2R}$ (negative sign signifies that the body is bound to the earth)

70. If kinetic energy is E, then potential energy will be – 2E and total energy will be – E.

71. When the altitude of the satellite increases, the potential energy will increase and kinetic energy will decrease.

72. a) The increase in gravitational potential energy of a body of mass 'm' taken to a height 'h' from the surface of the earth = $mgh\left(\frac{R}{R+h}\right) = \frac{GMmh}{R(R+h)}$.

b) Period of revolution of a satellite

$$= \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}$$

c) For a satellite going round the earth in a circular orbit at height h, the orbiting velocity = $\sqrt{\frac{gR^2}{R+h}}$

d) If the satellite is very close to the earth, then $T = 2\pi\sqrt{\frac{R}{g}}$ = 84.6 minutes.

e) If the satellite is very close to the earth, then $\omega = \sqrt{\frac{g}{R}} = 1.24\times10^{-3}$ rads^{-1}.

73. **Geo Stationary Satellite :**

a) An orbit in which the time period of revolution of a satellite is 24 hours is called **geostationary orbit** or **parking orbit** or **synchronous orbit.** It appears stationary with respect to the earth.

b) Radius of the geo–stationary orbit is approximately 42,400 km. Speed of geo–stationary satellite in it is 3.1 kms^{-1}.

c) The relative velocity of a geostationary satellite with respect to the earth is zero.

d) Height of the parking orbit is 36,000 km approximately from the surface of earth.

e) Geo stationary satellite orbits above the equator in the equatorial plane.

f) Geostationary satellites are used

i) to study the upper layers of atmosphere

ii) to forecast the changes in the atmosphere

iii) to know the shape and size of the earth

iv) to identify the minerals and natural resources present inside and on the surface of the earth

v) to transmit the T.V. programs to distant places

vi) to study the properties of transmission of radio waves in the upper layers of the atmosphere and

vii) to undertake extensive research work on the planets, satellites and comets etc., which are present in space.

e) Moon is natural satellite of earth.

74. **Escape velocity:**

a) The velocity which a body must be projected so that it never returns back or goes out of the earth gravitational field.

b) The escape velocity of a body on earth or on any planet is

$V_e = \sqrt{2gR}$ or $V_e = \sqrt{\frac{2GM}{R}}$

c) It depends upon

Mass M of the earth or planet

Radius R of the earth or planet

d) It is independent of mass of the body and angle of projection.

Its value on earth surface is 11.2Kms^{-1}

e) If r. m. s. velocity of gas molecules is equal or greater than escape velocity, then there will be no atmosphere.

f) Orbital velocity and escape velocity are related as

$V_e = \sqrt{2}\, v_0$ or $V_e / V_0 = \sqrt{2}$

f) When a body is projected with escape velocity its total energy is zero.

g) V_e on earth = 11.2 kms^{-1},

V_e on moon = 2.5 kms^{-1}

75. If the gravitational force varies inversely as the n^{th} power of distance R, then the orbital velocity $V \alpha R^{(1-n)/2}$ and the time period $T \alpha R^{(n+1)/2}$.

76. **Relation between velocity of projection and shape of the orbit :** When a body is projected with a velocity 'v' from any height in a horizontal direction, then

    a) If $v < v_o$, the body falls and hits the ground in a parabolic path.

    b) If $v = v_o$, the body revolves round the earth in a circular orbit.

    c) If $v > v_o$ and $v < v_e$, the body revolves round the earth in an elliptical orbit.

    d) If $v = v_e$, the body escapes into space in a parabolic path.

    e) If $v > v_e$, the body escapes into space in a hyperbolic path.

77. Both the escape velocity and the orbital velocity are independent of the mass of the body.

78. Launching speed is about 8.5 kms^{-1} for a satellite at 300 km above the ground.

79. For a body revolving around the earth to escape from the orbit, the velocity of the body must be increased by 41.4% and the kinetic energy should be increased by 100%.

80. By launching a rocket near the equator in west to east direction, advantage is to the extent of 0.45 kms^{-1} in the launching speed.

81. If the r.m.s velocity of gas molecules is equal to the escape velocity, then there won't be any atmosphere.

82. An astronaut in the space–craft feels weightless–ness. (since the reaction force exerted by the artificial satellite on the astronaut is zero.)