SIMPLE HARMONIC MOTION

Synopsis :

1. The motion that repeats itself after regular intervals of time is called **periodic motion**.
2. If a particle in the periodic motion moves to and fro over the same path, the motion is said to be vibrating or oscillating.

   e.g. : Oscillations of the balance wheel of a watch, stretched violin string, loaded spring etc.,
3. **Damped Oscillations:** Many oscillating bodies do not move back and forth between precisely fixed limits, because frictional dissipate the energy of motion. Such oscillations are called damped oscillations.

   Ex: Stretched violin string soon stops vibratory.
4. **Equilibrium Position:** the point at which no net force acts on the oscillating body is known as equilibrium position or **Mean position**.
   i) At the Mean position displacement of the body is Minimum.
   ii) At the Mean position velocity of the body is Maximum.
   iii) At the Mean position acceleration of the body is Minimum.
   iv) At the mean position P. E. of the body is Minimum.
   v) At the Mean position K.E. of the body is Maximum
5. **Extreme Position:** the point at which maximum force acts on the oscillating body is known as Extreme Position.
   i) At the extreme position displacement of the body is maximum.
   ii) At the extreme position velocity of the body is minimum.
   iii) At the extreme position acceleration of the body is maximum.
   iv) At the extreme position P. E. of the body is maximum
   v) At the extreme position K. E. of the body is minimum.
6. If a particle moves along a straight line with its acceleration directed towards a fixed point in its path and the magnitude of the acceleration is directly proportional to the displacement from its equilibrium position, then it is said to be in **simple harmonic motion**.
7. If the to and fro motion is along a straight line it is called linear SHM. If the displacement is measured in terms of angles then it is called angular SHM.
8. A particle in SHM has (a) variable displacement, (b) variable velocity, (c) variable acceleration and (d) variable force.
9. Examples for linear SHM are 1. Vertical oscillations of a loaded spring. 2. Oscillations of a paper boat on water waves. 3. Vibrations of a tuning fork.

   4. Oscillations of a simple pendulum with small amplitude etc.
10. Examples for angular SHM are
    i) The oscillations of a freely suspended magnet in the earth's magnetic field.
    ii) Oscillations of a torsional pendulum.
    iii) Oscillations of a balancing wheel in a watch.
11. The time taken for one complete vibration or oscillation is called **time period (T)**.
12. The number of oscillations or vibrations made per second is called **frequency (n)**.
13. The maximum displacement of the particle measured from the equilibrium position is called **amplitude (r or)**.

14. **Phase:** Phase at any instant is that which gives the state of the vibrating particle with respect to time in a specified direction with reference to a fixed point (Mean position)

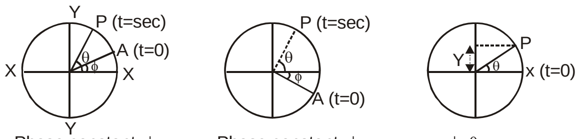
    Ex: (1) If the phase is zero, ie., the particle is crossing the mean position.

    (2) If the phase is $\pi/2$ ie., the particle is at the extreme position.

15. The initial phase at t = 0 of a particle in S. H. M. is called phase constant or epoch (ϕ).

    Ex: If $Y = A \sin(\omega t + \pi/4)$ at t = o $\phi = \pi/4$

16. **Representation of S. H. M.:** The motion of a particle executing periodic motion along a circular path is not simple harmonic motion. But the foot of the perpendicular dropped from the instantaneous position of a particle, executing periodic motion along any diameter is simple harmonic motion.



Phase constant=φ Y=Asinωt-φ

Phase constant=φ Y=Asinωt+φ

φ=0 Y=Asinωt

In general the simple harmonic motion is represented as $Y = A \sin(w t + \phi)$

Y = instantaneous displacement

A = Amplitude

Wt + ϕ = phase; ϕ is called initial phase.

i) If the motion starts from Mean Position $\phi = 0$

ii) If the motion starts from the extreme position $\phi = \pi/2$

17. Simple harmonic motion can be expressed by periodic functions like $A\sin\omega t$, $A\cos\omega t$, or combination of these functions.

    $Y = A \sin \omega wt + B \cos \omega t$

18. S.H.M. can also be represented in the following way ie., $F = ma$

    $$F = m.\frac{d^2y}{dt^2} = -KY \quad \text{or} \quad \frac{d^2y}{dt^2} + \left(\frac{k}{m}\right)y = 0$$

CHARACTERISTICS OF S. H. M.

19. **Instantaneous displacement:** The distance of the particle from mean position in a particular direction at any instant of time is known as instantaneous displacement.

    It is given by $Y = A \sin(\omega t + \phi)$

    If the particle starts from Mean position, $\phi = 0$ then

    i) $Y = 0$, at Mean position

    ii) $Y = A$ at extreme position

    iii) $Y = \frac{A}{2}$ at $\theta = \omega t = 30° = \pi/6$ or after a time interval $t = T/12$ sec.

    iv) $Y = \frac{A}{\sqrt{2}}$ at $\theta = \omega t = 45° = \pi/4$ or $t = T/8$ sec.

    v) $Y = \frac{\sqrt{3}}{2}A$ at $\theta = \omega t = 60° = \pi/3$ or $t = T/6$ sec.

20. **Velocity:** The rate of change of displacement is called velocity.

    $\therefore v = \frac{dy}{dt} = Aw\cos(wt + \phi)$

$v = \omega\sqrt{A^2 - y^2}$

If the particle starts from the mean position, $\phi = 0$ then

i) $v = A\omega$, ie., maximum at Mean Position

ii) $v = 0$, ie., minimum at extreme Position

iii) $v = \frac{A\omega}{2}$ at $\theta = \omega t = 60° = \pi/3$ or after a time interval of $t = T/6$ sec

iv) $v = \frac{A\omega}{\sqrt{2}}$ at $\theta = \omega t = 45° = \pi/4$ or after a time interval of $t = T/8$ sec

v) $v = \frac{\sqrt{3}}{2}A\omega$ at $\theta = \omega t = 30° = \pi/6$ or after a time interval of $t = T/12$ sec

21. **Acceleration:** the rate of change of velocity of a particle in S H M is called acceleration.

$\therefore a = \frac{dv}{dt} = -A\omega^2 \sin(\omega t + \phi) = -\omega^2 y$

$a \alpha - y$

If the particle starts from the Mean position, $\phi = 0$, then

i) $a = 0$, i.e., minimum at mean position

ii) $a = \omega^2 A$, i.e., maximum at extreme position.

iii) $a = \frac{\omega^2 A}{2}$ at $\theta = \omega t = 30° = \pi/6$, after a time interval of $t = T/12$ sec

iv) $a = \frac{\omega^2 A}{\sqrt{2}}$ at $\theta = \omega t = 45° = \pi/4$, after a time interval of $t = T/8$ sec

v) $a = \frac{\sqrt{3}\omega^2 A}{2}$ at $\theta = \omega t = 60° = \frac{\pi}{3}$, after a time interval of $t = T/6$ sec

22. The projection of uniform circular motion of a particle over any diameter of the circle is in SHM.

23. If two simple harmonic motions of same amplitude and frequency at right angles to each other are superposed, the resulting motion will be linear if the phase difference is 0 and circular if the phase difference is $\pi/2$ radians.

24. **Time Period:** Time taken by vibrating particle in S.H.M. to complete one vibration is called Time period or period of oscillation.

General formula :

$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$

i) $\therefore T = 2\pi\sqrt{\frac{y}{a}}$

ii) Time period of a simple pendulum =

$T = 2\pi\sqrt{\frac{l}{g}}$

l = length of the simple pendulum

g = acceleration due to gravity at a place.

iii) Time period of Torsion pendulum

$T = 2\pi\sqrt{\frac{I}{C}}$

I = moment of Inertia about the suspension wire C = couple per unit twist.

iv) Time period of a loaded spring

a) $T = 2\pi\sqrt{\frac{m}{k}}$

k = Force constant or spring constant

m = mass attached to the spring.

b) $T = 2\pi\sqrt{\frac{x}{g}}$

x = extension produced in the spring due to attachment of the load 'm'

g = acceleration due to gravity.

v) When a hole is drilled along the diameter of the earth and if a body is dropped in it, it moves to and from about the centre of the earth and is in S.H M. with a time period of

$T = 2\pi\sqrt{\frac{R}{g}}$ = 84.6 minutes or $T = \sqrt{\frac{3\pi}{GD}}$

D = Mean density of the earth

G = Gravitational constant

25. Kinetic energy at any instant

$= \frac{1}{2}m\omega^2 r^2 \cos^2 \omega t$

$= \frac{1}{2}m\omega^2 (r^2 - y^2)$

26. **Potential Energy :**

$P.E. = \frac{1}{2}m\omega^2 x^2 + U_0 = \frac{1}{2}m\omega^2 A^2 \sin^2 \theta + U_0$

Where m = mass of S.H.M.

x = displacement of S.H.M. from its mean position

A = amplitude of oscillation

θ = phase angle from its mean position

U_o= P.E. of S.H.M. at its mean position.

v) During one complete vibration average potential Energy is given by = $1/4\ m\omega^2 A^2$

27. **Kinetic Energy:**

i) The K. E. of a particle in S.H.M is given by K. E = $1/2\ m\omega^2 (A^2 - y^2)$

$= 1/2\ m\omega^2 A^2 \cos^2 (\omega t \pm \phi)$

ii) At mean position (y = o) K. E is maximum

iii) At extreme position (y = A) K. E. is zero.

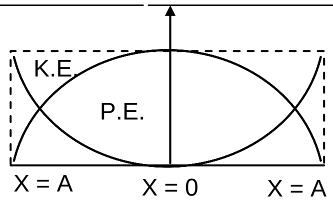
iv) During one complete vibration average kinetic Energy = $1/4\ m\omega^2 A^2$

28. **Total Energy:**

i) T. E. = P. E. + K. E. = $1/2\ m\omega^2 A^2 + U_0$

ii) When a particle is in S. H. M. At any position T. total energy is constant.

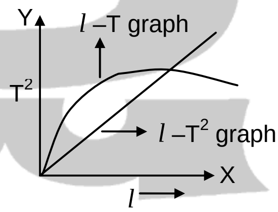
Energy and displacement curve.



29. In SHM, average kinetic energy = average potential energy = half of the total energy, when friction is zero.
30. For a body moving with SHM, velocity is 90° out of phase with the displacement and acceleration is 180° out of phase with the displacement. Velocity and acceleration have a phase difference of $\pi/2$ radians.
31. If n is the frequency of SHM, then the frequency with which kinetic energy or potential energy oscillates is 2n.

Simple pendulum:

32. The period of oscillation of a simple pendulum is independent of amplitude (for small values only), length being constant.
33. At constant length, the period of oscillation of a simple pendulum is independent of size, shape or material of the bob.
34. Time period of a simple pendulum (T) = $2\pi\sqrt{\frac{L}{g}}$.
35. Tension in the string of simple pendulum
    T_{min} = mg Cos θ (when bob is at extreme position)
    T = mg (3 – 2 Cosθ) (When bob is at any position)
    where θ is any angular amplitude.
36. $l - T^2$ graph of a simple pendulum is **straight line** passing through origin.
37. *l*–T graph of a simple pendulum is **parabola.**
38. At the point of intersection of *l*–T graph and *l*–T^2 graph of a simple pendulum



    i) T = 1 second
    ii) n = 1 Hz.
    iii) $l = \frac{g}{4\pi^2} \cong 25\text{cm}$ on the surface of the earth
39. **APPLICATION :**
    i) When length changes $\frac{T_1}{T_2} = \sqrt{\frac{L_1}{L_2}}$
    ii) For small percentage changes < 5%. $\frac{\Delta T}{T} \times 100 = \frac{1}{2}\frac{\Delta L}{L} \times 100$
    iii) For percentage $\geq$ 5%.If the percentage change is "n" then $\frac{\Delta T}{T} \times 100 = \left(2n + \frac{n^2}{100}\right)$
    iv) When the elevator is going up with an acceleration a, then its time period is given by $T = 2\pi\sqrt{\frac{L}{g+a}}$ and frequency n is given byn = $\frac{1}{2\pi}\sqrt{\frac{g+a}{L}}$
    v) When the elevator is moving down with an acceleration a, then its time period is given by $T = 2\pi\sqrt{\frac{L}{g-a}}$ and frequency n is given by
    $n = \frac{1}{2\pi}\sqrt{\frac{g-a}{L}}$

vi) When the elevator is at rest or moving up or down with constant velocity the time period is given by T = $2\pi\sqrt{\frac{L}{g}}; n = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

vii) When the elevator is moving down with an acceleration (–a) then its time period is given by T= $2\pi\sqrt{\frac{L}{g+a}}; n = \frac{1}{2\pi}\sqrt{\frac{g+a}{L.}}$

viii) In case of downward accelerated motion is a>g the pendulum turns upside and oscillates about the highest point with T = $2\pi\sqrt{\frac{L}{a-g}}$

ix) If a simple pendulum of length 'L' suspended in a car that is travelling with a constant speed around a circle of radius 'r', Then its time period of oscillation is given by

$$T = 2\pi\sqrt{\frac{L}{\sqrt{g^2 + \left(\frac{v^2}{r}\right)^2}}}$$

x) If a simple pendulum of length 'L' suspended in car moving horizontally with an acceleration 'a' is given by

$$T = 2\pi\sqrt{\frac{L}{\sqrt{g^2 + (a)^2}}}$$

The equilibrium position is inclined to the vertical by an angle 'θ'.

Where $\theta = \tan^{-1}\left(\frac{a}{g}\right)$ opposite to the acceleration.

xi) If the bob of a simple pendulum is given a charge 'q' and is arranged in an electric field of intensity 'E' to oscillate.

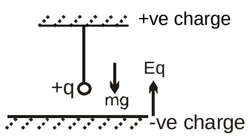
a) opposite to g, $\rightarrow$ Electric force E_q will be opposite to the force mg. Hence $g^1 = g - \frac{Eq}{m}$

Then $T_1 = 2\pi\sqrt{\frac{l}{g - \frac{Eq}{m}}}$. So time period increases.

b) In the direction of g $\rightarrow$ Electric force Eq will be in the direction of force mg. Hence

$g^1 = g + \frac{Eq}{m}$ then

$T_1 = 2\pi\sqrt{\frac{l}{g + \frac{Eq}{m}}}$



so time period decreases.

c) Perpendicular to g $\rightarrow$ Electric force E_q will be perpendicular to the force mg. Hence

$g^1 = \sqrt{g^2 + \left(\frac{Eq}{m}\right)^2}$ Then

$$T_1 = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \left(\frac{Eq}{m}\right)^2}}}.$$

So time period decreases.

xii) If a simple pendulum of length L is suspended from the ceiling of a cart which is sliding without friction on an inclined plane of inclination 'θ'. Then the time period of oscillations is given by T = $2\pi\sqrt{\frac{L}{g\cos\theta}}$ Since the effective acceleration changes from g to g cosθ.

xiii) Time period of a pendulum of length comparable to the radius of earth is
$T = 2\pi\sqrt{\frac{LR}{(L+R)g}}$.

xiv) The maximum time period of simple pendulum (pendulum of infinite length) is given by = $2\pi\sqrt{\frac{R}{g}}$ = 84.6 minute = 1.4 hour

xv) The time period of a simple pendulum having a length equal to the radius of the earth is $T = 2\pi\sqrt{R/2g}$ and is equal to $42.3\sqrt{2}$ minutes or 59min and 5 sec.

xvi) When a pendulum clock is taken from the equator to the poles the time period decreases. Hence it makes more oscillation and gains time and moves fast.

xvii) When a pendulum is taken from the earth to moon, the time period increases (as g is less on moon). Hence it makes less number of oscillations and looses time or moves slow.

xviii) When a pendulum clock is taken from the earth to the moon, to keep the time correct its length must be decreased.

xix) If the pendulum of a clock is made of metal, it runs slow during summer and fast during winter due to thermal expansion or contraction.

xx) If a boy sitting in a swing stands up, as centre of Mass rised up, length of the pendulum decreases and hence the period of the swing decreases.

xxi) If the bob of a pendulum is made hollow and filled with water, and the water is drained up as the water goes down, centre of mass shifts down, and then rises to its original position. Hence time period first increase and then decreases and reaches its original value.

40. If a pendulum clock is shifted from earth to moon then it runs $\sqrt{6}$ times slower.

41. For a simple pendulum at a place L/T^2 is a constant and $g = 4\pi^2 \frac{L}{T^2}$.

42. The tension in the string at any position is equal to $T = mg\cos\theta + \frac{mv^2}{l}$.

43. If a simple pendulum is arranged in an artificial satellite its time period becomes infinity.

44. Work done by the string of the simple pendulum during one complete vibration is equal to zero.

45. A simple pendulum fitted with a metallic bob of density d_s has a time period T. When it is made to oscillate in a liquid of density d_1 then its time period increases.

$$T = 2\pi\sqrt{\frac{l}{g\left(1-\frac{d_l}{d_s}\right)}}$$

46. When two simple pendulum of lengths l_1, and l_2, $l_2 > l_1$ are into vibration in the same direction at the same instant with same phase. Again they will be in same phase after the shorter pendulum has completed n oscillations. To find the value of n,
$nT_s = (n-1)\,T_l$ and $T \propto \sqrt{l}$

$$\therefore \frac{n}{n-1} = \frac{T_L}{T_S} \text{ or } \frac{n}{n-1} = \sqrt{\frac{l_L}{l_s}} \text{ or } n = \frac{1}{1-\sqrt{\frac{l_S}{l_L}}}$$

S – shorter L – Longer

47. **Seconds pendulum:**

i) The simple pendulum whose time period equal to 2 seconds is called seconds pendulum.

ii) its length at place where g = 9.8 m/s^2 is 100 cm.

iii) Since T = 2 sec

$$L = \frac{g.T^2}{4\pi^2} \Rightarrow L = \frac{g}{4\pi^2}.4$$

$$\therefore L = \frac{g}{\pi^2}$$

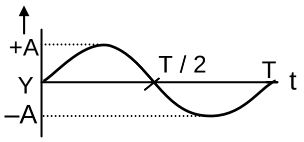
iv) The length of a pendulum at a place where $g = g_1$ is l_1 and its length at a place where $g = g_2$ is l_2 To keep the time period constant at T = 2 sec. its length has to be decreased or increased corresponding to the value of 'g' at that place

Decrease in length = $\frac{g_1 - g_2}{\pi^2}$ (if $g_1 > g_2$)

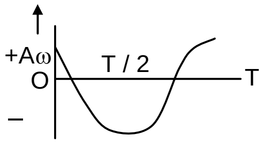
Increase in length = $\frac{g_2 - g_1}{\pi^2}$ (if $g_2 > g_1$)

48. A pendulum clock runs slow when i) L increases and ii) g decreases.
49. A pendulum clock runs fast when i) L decreases and ii) g increases.
50. A pendulum clock stops functioning at any place where the gravity is absent such as in an orbiting satellite, the centre of the earth the time period is infinity.
51. If the length of a simple pendulum is increased by x% (when x is very small), the period increases by x / 2 percent.
52. If the value of g increases by x%, the time period decreases by x / 2 percent.
53. If a wire of length L, area of cross–section A and Young's modulus Y is loaded by a mass m, the period of oscillation (T)= $2\pi\sqrt{\frac{mL}{YA}}$ or T= $2\pi\sqrt{\frac{x}{g}}$ where x is the elongation produced.
54. If a U–tube contains a liquid up to a vertical height h and the liquid in one limb is slightly pushed and released, the oscillation of liquid column is simple harmonic with a time period $2\pi\sqrt{h/g}$.
55. **GRAPHS:** (particle starts from mean position)

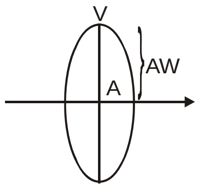
i) Displacement – time graph



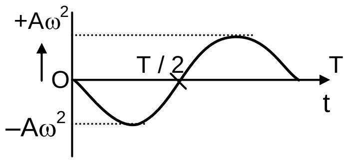
ii) Velocity – time graph



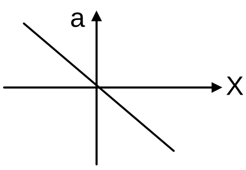
iii) Displacement – velocity graph



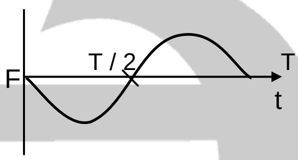
iv) Acceleration – time graph



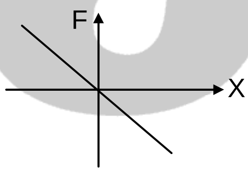
v) Acceleration – displacement graph



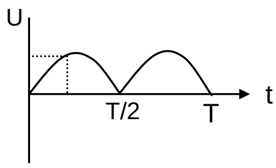
vi) Force – time graph



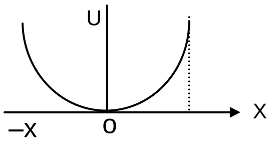
vii) Force – displacement



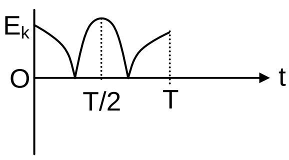
viii) Potential Energy – time graph



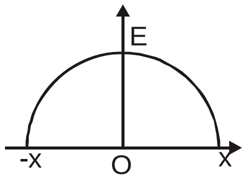
ix) Potential Energy – displacement graph



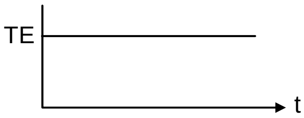
x) Kinetic Energy – time graph



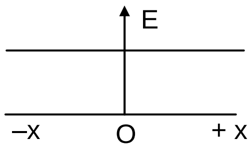
xi) Kinetic Energy – displacement graph



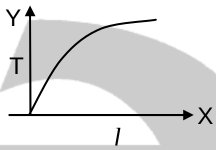
xii) Total Energy – time graph



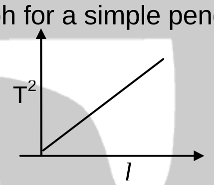
xiii) Total Energy – displacement graph



xiv) L – T graph for a simple pendulum



xv) L – T^2 graph for a simple pendulum.



SPRINGS :

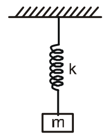
56. The spring constant of a spring may be defined as the force required to produce an extension of one unit in the spring. K = F / x.
57. Potential energy of the spring $= \frac{1}{2}Fx = \frac{1}{2}\frac{F^2}{K} = \frac{1}{2}Kx^2$
58. If a spring is cut into two pieces (of equal size), each piece will have a force constant double the original.
59. When a spring of force constant k is cut into n equal parts, the spring constant of each part is nk
60. If a uniform spring of spring constant K is cut into two pieces of lengths in the ratio $l_1 : l_2$, then the force constants of the two springs will be

    $K_1 = \frac{K(l_1 + l_2)}{l_1}$ and $K_2 = \frac{K(l_1 + l_2)}{l_2}$
61. The spring constant of a spring is inversely proportional to the number of turns.

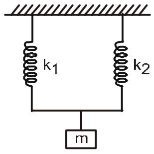
    F / x or Kn = constant or $K_1 n_1 = K_2 n_2$.
62. For a given spring greater the number of turns, greater will be the work done.

    $w \alpha n$

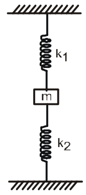
    $\frac{w_1}{w_2} = \frac{n_1}{n_2}$
63. If two springs of force constants k_1 and k_2 are joined in series, the combined force constant $k = \frac{k_1 k_2}{k_1 + k_2}$.
64. If two springs of force constants k_1 and k_2 are joined in parallel, the combined force constant $k = k_1 + k_2$.
65. When a body is just dropped on a spring, the maximum compression is double that of when the body rests on it in equilibrium.



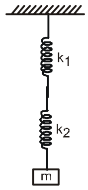
$$T = 2\pi\sqrt{\frac{m}{k}}$$



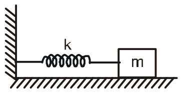
$$T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$



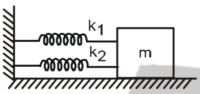
$$T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$



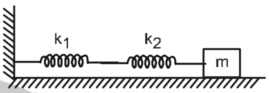
$$T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$



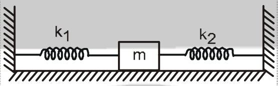
$$T = 2\pi\sqrt{\frac{m}{k}}$$



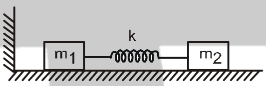
$$T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$



$$T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$



$$T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$



$$T = 2\pi\sqrt{\frac{m_1 m_2}{k(m_1 + m_2)}}$$