FLUID MECHANICS

Synopsis :

FLUID STATICS

1. A substance which can flow is known as **fluid**. Hence the term includes both liquids and gases.
2. The study of fluids at rest is called **fluid statics**.
3. Solids are incapable of flow because the intermolecular forces are very strong. Hence solids possess a definite shape and volume.
4. The intermolecular forces are weak in liquids. Hence liquids do not possess a definite shape but take the shape of the container.
5. In the case of gases, the intermolecular forces are practically non-existent. Therefore gases possess neither a definite shape nor a definite volume.
6. Density of a homogeneous substance is defined as the ratio of its mass to its volume. In other words density is the mass per unit volume.

   Mathematically, $d=\frac{m}{v}$, where d=density, m=mass and v=volume

   S.I unit of density is kgm^{-3}.
7. **Specific gravity** of a material is defined as the ratio of its density to that of water at 4^oC It is a mere number and has no units. It is also known as relative density.
8. **Specific gravity** of a substance can be said to be numerically equal to its density in grams/c.c.
9. If equal volumes of two liquids of densities d_1 and d_2 are mixed together, then the density d of the mixture is

   $$d=\frac{d_1+d_2}{2}$$
10. If equal masses of two liquids of densities d_1 and d_2 are mixed together, then the density 'd' of the resultant mixture is

    $$d=\frac{2d_1d_2}{d_1+d_2}$$
11. **Pressure** is defined as the ratio of the normal force acting on the area on which the force acts.

    $P=\frac{F}{A}$ where P=pressure, F=normal component of force and A = area on which force acts.

    S.I unit of pressure is pascal (Pa).

    Pressure is a scalar quantity.

Fluid Pressure :

12. The pressure due to a liquid column of height 'h' and density 'd' is given by

    P = hdg where g = acceleration due to gravity

    This is called **gauge pressure**.
13. The pressure exerted by the atmospheric air at any point is equal to the weight of air contained in a column of unit cross sectional area and extending up to the top of the atmosphere. This is called **atmospheric pressure**. Often it is expressed interms of the height of an equivalent mercury column (in a barometer).

14. The total pressure P acting at the bottom of an open liquid column of height 'h' and density 'd' is given by $P=P_a+hdg$ where P_a=atmospheric pressure
15. The total pressure P is called **absolute pressure**.
16. Absolute pressure=Gauge pressure + Atmospheric pressure
17. Atmospheric pressure=1.013×10^5 Pa
    =1.013×10^6 dynes/cm^2
    =76 cm of Hg
    =760 torr = 1.013 bar
18. If 'h' is the difference in heights of mercury in the two limbs of a **manometer**, then
    the gauge pressure=hdg and
    the total pressure = hdg + atmospheric pressure
19. In a U-tube or **Hare's apparatus**, if h_1 and h_2 are the heights of water and liquid columns (in balancing method) respectively, then the specific gravity of the liquid=$\frac{h_1}{h_2}$.
20. A **sphygmomanometer** is a type of blood pressure gauge commonly used by physicians.
21. **Pascal's law :** When ever pressure is applied on any part of a fluid contained in a vessel, it is transmitted undiminished and equally in all directions.
22. The **Bramah's press** works on the principle of Pascal's law. It is used to compress cotton bales, extract oil from seeds and drill holes in large metal sheets.
23. If in a hydraulic press (Bramah's press), the area of the smaller and larger pistons are 'a' and 'A' and a force 'f' is applied on the smaller piston, then the force 'F' developed on the larger piston is given by $F = \frac{fA}{a}$.
24. If we take 'n' containers each having the same base area and each containing the same liquid to the same height 'h', the pressure acting on the bases of each container is equal. This is known as **hydrostatic paradox or Masson's paradox**.
25. The pressure acting on the walls of a container having liquid in it is given by $\frac{1}{2}hdg$.
26. The pressure at a point in a liquid is same in all directions.
27. **Archimede's principle:** When a body is immersed wholly or partially in a fluid at rest, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.
28. The loss of weight (ΔW) of a solid when immersed in a liquid is given by $\Delta W=vdg$ where v=volume of the displaced liquid, d=density of the liquid and g=acceleration due to gravity.
29. From Archimedes' principle
    i) relative density of solid = $\frac{\text{weight of solid in air}}{\text{loss of weight of solid in water}}$
    ii) relative density of a liquid = $\frac{\text{loss of weight of sinker in liquid}}{\text{loss of weight of sinker in water}}$
30. Some applications of Archimede's principle are
    i) hot air (or helium) balloon

ii) determination of purity of precious metals like gold, silver etc.

iii) submarines, etc.

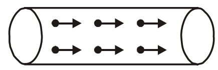
31. The upward force is called the **buoyant force** or **force of buoyancy**.
32. Buoyant force depends on the volume of the displaced liquid and not on the volume of the body.
33. Buoyant force depends on the density of the liquid and not on the density of the body.
34. When a body is immersed in a fluid, when
    i) if the weight of the body (W) is more than the upthrust (W^l) i.e., $W>W^l$, the body will sink.
    ii) if the weight of the body (W) is equal to upthrust (W^l) i.e., $W=W^l$, the body will float, the whole if its volume being inside the liquid and
    iii) if the weight of the body (W) is less than the upthrust (W^l) i.e., $W<W^l$, the body will float with a part of it being outside the liquid.
35. The buoyant force acts vertically upwards through the center of gravity of the displaced liquid.
36. The center of gravity of the displaced liquid is called **center of buoyancy**.
37. When a body floats the vertical line joining the center of buoyancy and the center of gravity of the body is called **central line**.
38. When a floating body is slightly disturbed, the point where the vertical line from the center of buoyancy intersects the central line is called **meta center**.
39. If the meta center lies above the center of gravity, the body will remain in stable equilibrium.
40. If the meta center lies below the center of gravity, the body will be in unstable equilibrium.
41. If the meta center coincides with the center of gravity, the body will be in neutral equilibrium.
42. When a solid of density 'ρ' floats in a liquid of density 'd', then the volume fraction of solid immersed in liquid is given by

    $V_i=\frac{\rho}{d}$ where V_i=volume fraction of the solid inside the liquid.

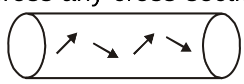
    $V_o=1-V_i$ where V_o=volume fraction of the solid outside the liquid.
43. When an ice block floating on water melts, the level of water remains the same.
44. If a floating piece of ice contains an air bubble, the level of water does not change when the ice metals.
45. If a floating ice block contains a piece of cork embedded inside, there is no change in the level of water when the ice melts.
46. A floating block of ice contains a piece of lead. The level of water decreases when the ice melts.
47. When a block of ice floating on a liquid denser than water melts, there is an increase in the level of the liquid.
48. When a block of ice floating on a liquid whose density is less than that of water melts, there is a decrease in the level of the liquid.
49. A man is sitting in a boat which is floating in a pond. If the man drinks some water from the pond, the level of the water remains the same.
50. A boat carrying a number of stones is floating in a water tank. If the stones are unloaded into the water, the water level in the tank decreases.
51. A **hydrometer** is used to measure the density of a liquid directly.
52. A **lactometer** is used to determine the purity of milk.

FLUID DYNAMICS :

53. The study of fluids in motion is called fluid **dynamics**.
54. Fluids flow from one place to other because of pressure differences.
55. **Streamline flow :** The flow of a liquid is said to be streamlined or orderly if the particles of the liquid move along fixed paths known as streamlines and velocity of the particles passing one after the other through a given point on a streamline remains unchanged in magnitude as well as direction at that point.



56. Streamline in flow is also called **laminar flow** or **steady flow**.
57. A streamline in general follows the shape of the tube through which the liquid flows. Thus it may be straight or curved.
58. Steady flow or streamline flow :
    i) Streamline flow is that flow in which every particle flows along the path of its preceding particle.
    ii) The path taken by a particle in a flowing fluid is called its **line of flow**.
    iii) The tangent at any point on the line of flow gives the direction of motion of that particle at that instant.
    iv) Streamlines may be straight or curved.
    v) Two streamlines cannot intersect each other.
    vi) There is no radial flow in the tube.
    vii) The mass of fluid entering the tube in unit time is equal to the mass of fluid leaving the tube in unit time.
    viii) Pressure over any cross-section is constant.
    ix) The velocity at any point of the liquid remains the same throughout the time for which the flow is maintained.
    x) The energy supplied to the fluid for maintaining its flow is mainly used in overcoming the viscous drag between different layers.
59. **Tube of flow :** A bundle of streamlines having the same velocity of fluid elements over any cross-section perpendicular to the direction of flow is known as tube of flow.
60. **Turbulent flow :** If the velocity of a point of a fluid varies in time it is called turbulent flow. In turbulent flow the liquid flows in a disorderly fashion growing eddies and vortices.
61. **Rate of flow :** The rate of flow of a liquid is the volume of a liquid that flows across any cross-section in unit time and is given by



$$Q=\frac{\text{volume}}{\text{time}}\ m^3/s$$

$$Q=AV$$

where A is the area of cross-section of the tube V is velocity of the liquid

62. **Principle of continuity :** In case of steady flow of incompressible and non viscous fluid through a tube of non-uniform cross-section, the product of the area of cross-section and the velocity of the flow is same at every point in the tube.

    $A \times V$ =constant

    $A_1V_1=A_2V_2$ It is called **equation of continuity**
63. Equation of continuity represents the law of conservation of mass in case of moving fluids.

64. **Total energy of a liquid :** The total energy at any point in a flowing liquid is of three kinds. (a) potential energy, (b) kinetic energy, (c) pressure energy.

a) **Potential energy :** The energy possessed by a liquid by virtue of its height above some arbitrary level is called potential energy.

Potential energy of mass m of the liquid=mgh

Potential energy per unit mass m=gh

b) **Kinetic energy :** The energy possessed by a liquid by virtue of its motion is called kinetic energy.

Kinetic energy per unit mass= $\frac{1}{2}v^2$

c) **Pressure energy :** The energy possessed by a liquid by virtue of the pressure acting on it is called pressure energy.

Pressure energy in volume dw = PdV

Pressure energy per unit mass = $\frac{P}{\rho}$

Where ρ is density of liquid.

65. **Bernoulli's theorem :** If an ideal fluid (non viscous, incompressible) is in streamline flow in a tube of non uniform cross-section the sum of the pressure energy, kinetic energy and potential energy at any point per unit mass or per unit volume is constant.

$$\frac{P}{\rho}+\frac{V^2}{2}+gh=\text{constant}$$

$$\frac{P}{\rho g}+\frac{V^2}{2g}+h=\text{constant}$$

here $\frac{P}{\rho g}$=pressure head

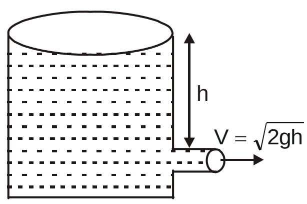
$\frac{V^2}{2g}$=velocity head

h=gravitational head

$$P+\rho gh+\frac{1}{2}\rho V^2=\text{constant}$$

$P \rightarrow$ is called static pressure

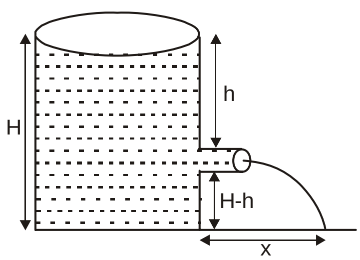
$\frac{1}{2}\rho V^2 \rightarrow$ dynamic pressure



Applications of Bernoulli's theorem :

66. **Torricelli's theorem :** The velocity of efflux of a liquid through an orifice (small hole) of a vessel is equal to the velocity acquired by a freely falling body from a height which is equal to that of liquid level from the orifice.

$V=\sqrt{2gh}$



67. Time taken by the efflux liquid to reach the ground is given by $t=\sqrt{\frac{2(H-h)}{g}}$

68. Horizontal range of liquid is given by

$x = \sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}} = 2\sqrt{h(H-h)}$

69. Horizontal range is maximum when orifice is at the middle of liquid level and bottom.

$h = \frac{H}{2}$; $x_{max} = H$

The horizontal range (x) of liquid coming out of the holes at depths h or (H–h) from its free surface is the same.

70. Time taken for the level to fall from H_1 to H_2

$t = \frac{A}{A_o}\sqrt{\frac{2}{g}}\left[\sqrt{H_1} - \sqrt{H_2}\right]$

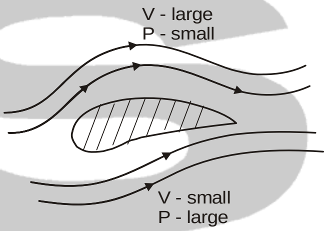
where A_o is area of orifice, A is area of cross-section of container.

71. If the hole is at the bottom of the tank, time t taken to emptied the tank

$t = \frac{A}{A_o}\sqrt{\frac{2H}{g}}$

Dynamic Lift :

72. The upward lift experienced by a body in motion when immersed in a fluid is called **dynamic lift**.
73. The dynamic lift experienced by a body when it is in motion in air is called **aerodynamic lift**.
74. **Lift on an Aircraft wing :** Aeroplane wings are so designed (i.e., streamlined) the velocity of air flow above the wing is higher than the velocity of air flow under the wing. This difference of air speeds, in accordance with Bernoulli's principle, creates pressure difference, due to which an upward force called "dynamic lift" acts on the plane.

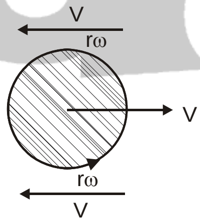


Dynamic lift=pressure difference x area of the wing

$= (P_2 - P_1) \times A$

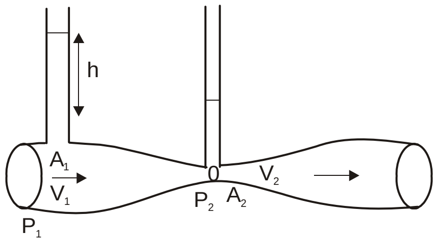
$= \frac{1}{2}\rho\left[V_2^2 - V_1^2\right] \times A$

75. **Magnus effect :** When spinning ball is thrown, it deviates from its usual path in flight. This effect is called Magnus effect and plays an important role in tennis, cricket and soccer etc.



76. If the ball is moving from left to right and also spinning about a horizontal axis perpendicular to the direction of motion, then relative to the ball air will be moving from right to left. The resultant velocity of air above the ball will be $v+r\omega$ while below it $v-r\omega$. So in accordance with Bernoulli's principle pressure above the ball will be less than below it. Due to this difference of pressure an upward force will act on the ball and hence the ball will deviate from its usual path.
77. If ball is thrown with back spin, the pitch will curve less sharply prolonging the flight.
78. If the spin is clockwise, the pitch will curve more sharply shortening the flight.
79. When wind blows over a house with high speed pressure on the roof will be less that of inside the house and so the roof is lifted and blown away by the wind.
80. **Atomizer** (sprayer); paint gun and Bunsen burner, pilot tube, carburetor, filter pump (aspriator) work n basis of Bernoulli's principle.

81. **Venturimeter** : Venturimeter is used to measure flow speed and rate of flow in a pipe.

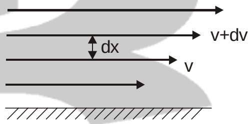


Velocity of flow $V_1 = \sqrt{\dfrac{2gh}{\left(\dfrac{A_1}{A_2}\right)^2 - 1}}$

Rate of flow $Q = A_1\sqrt{\dfrac{2gh}{\left(\dfrac{A_1}{A_2}\right)^2 - 1}}$

VISCOSITY :

82. When water flows in a uniform horizontal tube there is fall in pressure along the tube in the direction of flow. The reason for this fall in pressure is that force is required to maintain motion against friction. This friction is nothing but viscous forces of the liquid.



83. When liquid flows, it can be assumed to be composed of different horizontal layers. The upper layer tries to drag forward due to cohesive forces between the molecules of adjacent layers) the lower layer increasing the velocity of lower layer whereas the lower layer tries to drag backward the upper layer decreasing the upper layer's velocity. So there exists a velocity gradient perpendicular to the plane of liquid.
84. The velocity of the layers goes on decreasing as the depth increases and finally the deepest layer in contact with the horizontal surface is at rest.
85. **Viscosity :** The property of a fluid which opposes the relative motion between different layers is called viscosity.
86. Viscosity is the internal resistance or friction exhibited between the layers of a fluid.
87. **Viscous force :** (Newton's formula) The viscous force (F) acting tangentially on a layer of a fluid is directly proportional to the (i) surface area A of the layer, (ii) velocity gradient $\left(\dfrac{dV}{dx}\right)$ which is perpendicular to the direction of flow.

    $F = -\eta A\dfrac{dV}{dx}$

    This law is called Newton's law of viscous flow in streamline motion. The constant of proportionality η is called the coefficient of viscosity.
88. **Coefficient of viscosity "η" :** The viscous force acting tangentially on unit area of the liquid when there is a unit velocity gradient in the direction perpendicular to the flow is called the coefficient of viscosity. It is also called **coefficient of dynamic viscosity**.

The S.I unit of coefficient of viscosity is $\frac{N-S}{m^2}$ or Pa-s or decapoise.

The CGS unit of η is $\frac{dyne-s}{cm^2}$ or poise.

1 Pa-s=10 poise

The dimensional formula of η is $ML^{-1}T^{-1}$

For ideal liquid (non viscous, incompressible) η=0.

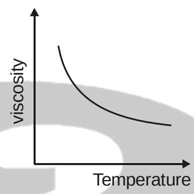
89. **Coefficient of kinematic viscosity "μ" :** The ratio of the coefficient of dynamic viscosity to the density of liquid $\left(\frac{\eta}{\rho}\right)$ is called coefficient of kinematic viscosity. It is often used by the mathematicians and engineers in fluid dynamics.

a) Dimensional formula L^2T^{-1}

b) The S.I unit of 'μ' is m^2s^{-1}

c) The unit in CGS system is stoke and 1

stoke=10^{-4} m^2s^{-1}



90. **Effect of temperature :** In case of liquids, coefficient of viscosity decreases with increase of temperature as the cohesive forces decrease with increase of temperature.

91. In the case of gases, coefficient of viscosity increases with increase of temperature because the change in momentum of molecules increases with increase of temperature.

92. **Effect of pressure :**

a) For liquids the value of η increases with increase of pressure.
Viscosity of water decreases with increase of pressure.

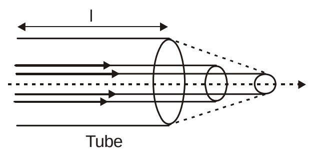
b) For gases, the value of η increases with increase of pressure at low pressure. But a high pressure η is independent of pressure.

93. The quality of fountain pen ink depends largely on its viscosity. The normal circulation of blood through our arteries and veins depends on the viscosity of blood. The shape of ship or the car is streamlined to minimize the effects of force of viscosity.

94. The cloud particles come down slowly and appear floating in space due to the viscosity of air.

95. The cause of viscosity in liquids is the cohesive forces among molecules whereas in gases it is due to diffusion.

96. **Poiseuille's equation :**



a) When a liquid flows through a capillary tube with streamline motion, the velocity of the liquid layer along the axis of the tube is maximum and gradually decreases as we move towards the walls where it becomes zero.

b) The volume of liquid flowing per second (rate of flow Q) through the tube depends on the following factors

i) the viscosity of liquid (η)

ii) the radius of the pipe

iii) the pressure gradient $\left(\frac{P}{l}\right)$ where P is pressure difference across the length l of the capillary tube.

$$Q=\frac{\pi}{8}\frac{Pr^4}{\eta l}$$

This formula is known as **Poiseuille's formula**.

This is applicable under the following conditions.

1) The flow must be steady and laminar.

2) The liquid in contact with the walls of the capillary tube must be at rest.

3) The pressure at any cross-section of the capillary tube must be same.

97. When a liquid is flowing through a tube, the velocity of the flow of a liquid at a distance x from the axis of the tube is given by $V=\frac{P}{4\eta l}\left[r^2-x^2\right]$.

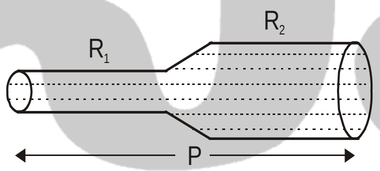
The velocity distribution curve of the advancing liquid in a tube is a parabola.

98. Ohm's law in fluid dynamics (Poiseuilli's equation)

a) The Poiseuilli's formula $Q=\frac{\pi}{8}\frac{Pr^4}{\eta l}$ can be written as $Q=\frac{P}{R}$ similar to Ohm's law i=V/R

where $R=\frac{8\eta l}{\pi r^4}$ is called fluid resistance.

b) when two capillaries are joined in series across constant pressure difference P the fluid resistance $R=R_1+R_2$.



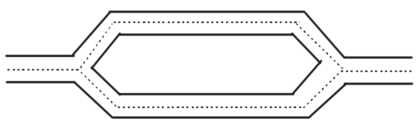
$$R=\frac{8\eta l_1}{\pi r_1^4}+\frac{8\eta l_2}{\pi r_2^4} \text{ and } Q=\frac{P}{R_1+R_2}$$

c) When two capillaries are joined in series, the rate of flow is the same but the pressure difference across the two tubes is different.

The total pressure difference $P=P_1+P_2$

d) If two capillaries are joined in parallel, the pressure difference across the two tubes is the same but the volume of fluid flowing through the two tubes is different. The total volume of the fluid flowing through the tubes is one second is $Q = Q_1 + Q_2$.

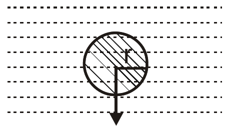
When two capillaries are joined in parallel across a constant pressure difference P, then fluid resistance R is given by $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$ where $R_1=\frac{8\eta l_1}{\pi r_1^4}$ and $R_2=\frac{8\eta l_2}{\pi r_2^4}$



The volume of liquid flowing through first capillary $Q_1=\frac{P}{R_1}$ and the volume of liquid flowing through second capillary $Q_2=\frac{P}{R_2}$.

99. Motion of objects through viscous medium :

a) **Stoke's law :** According to this law the viscous force acting on a freely falling smooth, spherical body is directly proportional to



i) coefficient of viscosity of fluid (η)

ii) radius of the spherical body (r)

iii) velocity of the body (V)

$F_\eta=6\pi\eta rV$. This is called Stoke's formula.

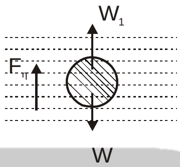
This is true for small values of V in a large expansion of fluid.

b) when a sphere falls vertically through a viscous fluid it is subjected to the following forces

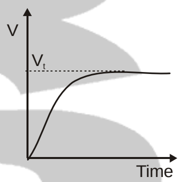
i) Its weight W acts downward

ii) The viscous resistance F_η acts upwards

iii) The Archimedes upthrust of buoyancy W_1 acts upwards.



c) **Terminal velocity :** When a body is dropped in a fluid at one stage the resultant force acting on it will be zero and it travels with uniform velocity and this is called terminal velocity.



If V_t represents the terminal velocity and F the maximum viscous resistance, then $F=W-W_1$

$$6\pi\eta rV_t=\frac{4}{3}\pi r^3(d-\rho)g$$

$$V_t=\frac{2}{9}\frac{gr^2(d-\rho)}{\eta}$$

Where d=density of the body, ρ=density of the liquid

d) Terminal velocity is directly proportional to square of the radius of the sphere, difference of densities and inversely proportional to the coefficient of viscosity of the fluid.

100. **Critical velocity :** The minimum velocity at which a liquid flow changes from streamline to turbulent flow is called critical velocity.

a) The critical velocity (Vc) of the fluid depends on i) viscosity (η), ii) the diameter of tube (D) through which the fluid is flowing, iii) density of the fluid ρ.

$$V_c=R\frac{\eta}{D\rho}$$

where R is a constant of proportionality and is called Reynold's number. It has no dimensions.

b) For a laminar flow, the value of R lies between 0 and 1000.

c) For values of R>2000, the flow will be turbulent.

d) For values of R between 1000 and 2000 the flow is unstable and switches from laminar flow to turbulent and vice versa.

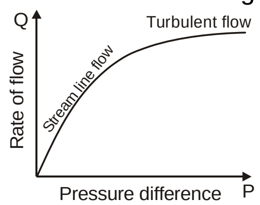
e) The orderly flow or streamline flow is produced when we use narrow tubes and liquids of low density and high viscosity.

f) Reynold's number is the ratio of force of inertia to force of viscosity.

101. In laminar flow, viscous force is proportional to the velocity.

102. In turbulent flow, viscous force is proportional to the square of velocity.

103. The bodies of air planes, torpedoes, ships, bombs and automobiles are streamlined to avoid wastage of energy in movement through the fluid. The man riding a race bicycle bends his body forward in order to have a streamlined shape.



104. A small value of Reynold's number means that the viscous forces predominate whereas the larger values of it indicate that the forces of inertia predominate.

105. In streamline flow, rate of flow is proportional to pressure difference. In turbulent flow; rate of flow is proportional to $\sqrt{P}$ approximately.