CYCLOIDAL CURVES

The curve obtained by a locus of a point on a circle when it roles on a straight line or on another circle above or below without slipping is a Cycloidal curve.

Let circle is a wheel here; Single wheel is rolling on a road.

P is mark on the wheel, when the wheel is moving from 1st position to 2nd position

1. The Mark P, position with respect to ground is changed.
   But the distance between the centers and mark P doesn’t change.(i.e, Radius need to take to locate P from corresponding centers)

2. The height of the centers from ground doesn’t change.

- Cycloidal curve obtained when circle rolls on straight line is called CYCLOID.
  E.g.: A coin rolls on a horizontal table.
      A man rides a bike on a road.
  Terms: rolling circle/ generating circle (radius is r)

  Base line (length is PQ, which is equal to \(2\pi r\) for 1 rev)

- Cycloidal curve obtained when circle rolls on another circle is called EPICYCLOID.
  Terms: rolling circle/ generating circle (radius is r)
  Base circle / directing circle (Radius is R)
• Cycloidal curve obtained when circle rolls inside another circle is called HYPOCYCLOID.
E.g.: A circus man rides a bike inside a globe.

Terms: rolling circle/ generating circle
Base circle or Directing circle

How to solve the Cycloidal problems:

For any problem, know the

1. Which Cycloidal curve?
   E.g.: A coin rolls on a horizontal table. → Cycloid
   A circus man rides a bike inside a globe. → Hypocycloid
   A man rides a bike on a road → Cycloid

2. How many revolutions ?(by default 1 rev)
   E.g.: One revolution [OR] 1 rev
       One and half revolution [OR] 1.5 rev [OR] \(1\frac{1}{2}\) rev
       One half revolution [OR] 0.5 rev

3. Where is the locus of point? (by default it is at point of contact)
   E.g.: The locus of point is diametrically opposite to the point of contact.

QUESTIONS
1. A circle 45mm diameter rolls along a straight line without slipping. Draw a curve traced out by point ‘P’ for

   a. One revolution [OR] 1 rev

   b. One and half revolution [OR] 1.5 rev [OR] \(1\frac{1}{2}\) rev

   Draw a tangent and normal to curve at distance of 35mm from straight line.

   a) One revolution [OR] 1 rev
**Drawing Procedure:**

1. Draw a rolling circle of diameter 45 mm.
2. Draw a PQ line at P, which is equal to circumference of the circle.
3. Divide the circle into 12 parts.
4. Give the numbering in CCW direction as 1, 2, 3, ..., since the circle is rolling in CW direction.
5. Draw horizontal lines through 1, 2, 3, ....
6. Divide the PQ into 12 parts (same as circle) as 1', 2', ....
7. Draw perpendicul ars from these points. These perpendicular lines meet the C line at c1, c2, ...
8. Now always radius is r and centre is c1 cut the horizontal line coming from the point 1 and locate p1.
9. Similarly take always radius is r and centre is c2 cut the horizontal line coming from the point 2 and locate p2.
10. Similarly get the point’s p3, p4, ..., p12.
11. Draw a smooth curve (free hand) through p1, p2, ....
Note:

1. Circle can be divided into 8 or 12 parts.
2. Locate c1, c2, ... etc., on C horizontal line only, but in this case C horizontal line is overlapping with line horizontal lines drawn from 3 and 9.
3. See that the curve should pass through the point P.

Tangent and normal for a cycloid:

Case 1: Draw a tangent and normal to curve at distance of 35mm from straight line.
Draw a parallel line at a distance of 35 mm to the straight line. The intersection of the drawn line and curve is the required point.

Case 2: Draw a tangent and normal to curve at distance of 35mm from circle.
For required point, cut the curve with the radius is 35mm and C as centre.

Drawing Procedure:

1. The point D as centre and radius equal to radius of the rolling circle cut the horizontal line drawn from C at C’.
2. Draw perpendicular line to base line form C’ and get E.
3. Join E and D, extend \( \rightarrow \) normal
4. Perpendicular to normal at that point is tangent.
(b) For one and half revolution [OR] 1.5 rev [OR] \(\frac{3}{2}\) rev

2. Draw a cycloid of a circle 45mm diameter. The locus of P is diametrically opposite to the point of contact.

3. Draw an Epicycloid of rolling circle 40 mm (2r) which rolls out side another circle of 150mm diameter for one revolution. Draw a tangent and normal at any point on the curve.
Drawing Procedure:

1. Calculate \( \theta = \frac{r}{R} \times 360^\circ = 96^\circ \)
2. Take any point O as centre and radius R, draw an arc PQ which subtends an angle \( \Theta = 96^\circ \).
3. Extend OP and mark C at a distance of radius r from P.
4. C as centre, CP as radius, draws a rolling circle.
5. Divide the circle into 8 or 12 parts. say 8 parts. give numbering as 1, 2, 3, ... 
6. Draw arcs from 1, 2, 3...8 with O as centre. And also draw an arc from C. i.e. O as centre OC as radius.
7. Divide OC arc in to 8 parts as that of rolling circle. Name as c1, c2……c8.
8. Now c1 as centre, r as radius mark p1 on a arc drawn from 1.
9. C2 as centre, r as radius mark p2 on a arc drawn from 2.
10. Similarly mark p3, p4...p8.
11. Join with a smooth curve (free hand) through these points.

Note:

1. C1, c2 .... Mark on C arc only. Don’t mark c1, c2 .... On 6, 2 arc.
Tangent and normal for an Epicycloid:

Drawing Procedure:
1. Locate M as per the given data.
2. With r as radius, M as centre mark N on C arc.
3. Join O and N.
4. The line ON intersect the base circle at S.
5. Join S and M, extend, the Normal.
6. Draw perpendicular to Normal i.e. tangent
4. Draw an Epicycloid of rolling circle 40mm (2r) which rolls outside another circle of 150mm diameter for one revolution. The locus point is diametrically opposite to the point of contact.

5.
6. Draw a hypocycloid of a circle of diameter 40mm which rolls inside another circle without slipping of 200mm for one revolution. Draw a tangent and normal at any point on it.

Note: Calculate \( \theta = \frac{r}{R} \times 360^\circ = 72^\circ \)