DUAL NATURE OF RADIATION AND MATTER

Important Points:

1. J.J. Thomson and Sir William Crookes studied the discharge of electricity through gases. At about 0.01 mm of Hg and at high voltage invisible streams called cathode rays are emitted. J.J. Thomson called them “streams of negative corpuscles”. Johnston Stony suggested the name “Electron”. Thus the electron was discovered.

2. **Photo Electric Effect:**

. When light of suitable frequency is incident on metal surface, electrons are emitted from that surface. This phenomenon is called photo-electric effect.

The minimum energy required to remove a photo electron from a metal surface is called work function of the metal (W).

$$W = h\upsilon_0$$

$$W = h\frac{c}{\lambda_0} = \frac{12400}{\lambda\left(A^0\right)}eV$$

3. **Einstein’s Photo Electric Equation**

$$E = W + KE$$

$$h\upsilon = W + \frac{1}{2}mv^2$$

$v \rightarrow$ Maximum velocity of emitted photoelectron

a. The minimum negative potential given to the collector. At which photoelectric current becomes zero is called stopping potential (V_0).

The above equation can also be written as $h\upsilon = h\upsilon_0 + eV_0$

4. Laws of Photo-Electric Effect:

a. This is instantaneous phenomenon

b. $K.E_{\max}$ is independent of intensity of incident radiation but directly proportional to frequency of incident radiation.

c. The photo electric current is independent of frequency of incident radiation but directly proportional to intensity of incident radiation.

d. The frequency of incident radiation of below the threshold frequency of metal no photo electrons are emitted.

5. De Broglie Hypotheses:

The waves associated with moving particles are called matter waves (or) de-Broglie waves. De Broglie predicted that the wavelength of these waves is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mev}} = \frac{12.29}{\sqrt{v}} A^0$$

6. Photon has both wave and particle nature. Its rest mass is zero.

7. The wave nature of electron can be explained by Davisson and Germer Experiment.

Very Short Answer Questions

1. What are “Cathode Rays”?

A. Cathode rays are a stream of negatively charged particles produced in a discharge tube at low pressure about 0.001 mm of Hg and high potential difference above10 KV.

2. What is the important fact did Millikan’s experiment establish?

A. Importance of Millikan’s Experiment:

Charge is always quantized. The charge on any particle is only an integral multiple of the charge of the electron.

3. What is “Work Function”?

A. The minimum amount of energy required just to eject an electron from the surface of a metal is called the work function of the metal. It is generally denoted by ϕ_0 and measured in eV.

Work function. $W_0 = h\nu_0 = \frac{hc}{\lambda_0}$ Where ν_0 is threshold frequency and λ_0 is Threshold wavelength.

4. What is Photo Electric Effect?

A. Photo electric effect is the phenomenon of emission of electrons by metals when illuminated by light of suitable frequency.

5. Give examples of “Photosensitive Substances”. Why are they called so?

A. Lithium, Sodium, Potassium, Cesium and Rubidium are some examples of photo sensitive metals, because the work function of these metals is very low and they are very sensitive even to visible light also to emit electrons.

6. Write down Einstein's Photo Electric Equation?

A. Photon energy is utilized in two ways. Some part of energy is utilized to remove the electron from the metal surface (Work Function W_0) and remaining part of energy appears as kinetic energy of an electron.

Photon energy = work function + kinetic energy.

$$h\upsilon = W_0 + \left(\frac{1}{2}m\upsilon^2\right)_{max}$$

7. Write down the de-Broglie's relation and explain the terms therein?

A. According to Louis de Broglie's hypothesis, every moving particle will have some wave associated with it. They are called matter waves or de Broglie Waves.

The ratio between the Planck's constant and the momentum of the particle is called De Broglie Wavelength.

$\lambda = \frac{h}{p} = \frac{h}{mv}$ Where v = velocity of the particle and h = Planck's constant.

8. State Heisenberg's Uncertainty Principle?

A. Heisenberg's Uncertainty Principle:

According to Heisenberg's Uncertainty Principle, it is impossible to measure simultaneously both the position and the momentum of the particle.

Let Δx and Δp be the uncertainty in the simultaneous measurement of the position and momentum of the particle, then $\Delta x \Delta p = \hbar$; where $\hbar = \frac{h}{2\pi}$ and h = 6.63 $\times 10^{-34}$ *J-s* is the Planck's constant ($\frac{h}{2\pi} = 1.05\times10^{-34}\,J-\sec$).

Short Answer Questions

1. What is the effect of (i) Intensity of Light (ii) Potential on Photoelectric Current?

A. (i) Effect of Intensity of Light:

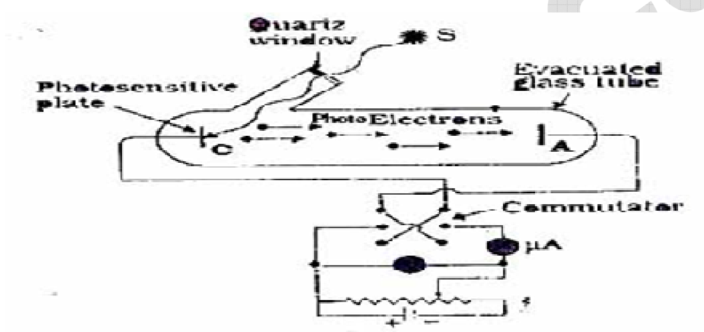
The number of photoelectrons emitted per second (photoelectric current) is proportional to the intensity of the incident radiation.

(ii) Effect of Potential:

For a given frequency of incident radiation, the stopping potential is independent of intensity of the incident radiation. Hence the number of photoelectrons emitted per second (photoelectric current) is independent potential.

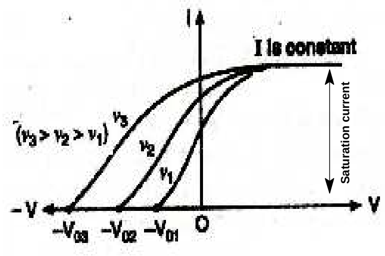
2. Describe an experiment to study the effect of frequency of incident radiation on 'Stopping Potential'?

A.



When a monochromatic light of suitable frequency from the source S after being filtered by a filter attached to a window W, fall on photosensitive plate C (emitter), photo electrons are emitted these electrons are accelerated towards the collector plate A .The emission of electrons causes flow of current called photo electric current in the circuit . Micro ammeter measures photo electric current. The plate A can be maintained at desired positive or negative potential with respect to emitter by using a battery.

By taking radiations of different frequencies but of same intensity, the variation of photo electric current with potential is shown in below graph.



From Graph:

1) The value of stopping potential is different for radiations of different frequencies.

2) Greater the frequency of incident light, greater is the stopping potential.

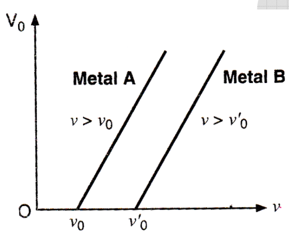
$$V_{03} > V_{02} > V_{01}$$

3) Value of saturation current depends on intensity but not on frequency of radiation.

υ and V_0 Graph:

1) For a given photo sensitive material, stopping potential varies linearly with frequency of radiation.

2) For a given photo sensitive material, there is certain minimum cut off frequency for which stopping potential equal to zero.



Since $\upsilon_0' > \upsilon_0$, the threshold frequency is more for metal B than for metal A.

3. Summarize the photon picture of electromagnetic radiation?

A. Photon picture of Electromagnetic Radiation:

(i) In interaction of radiation with matter, radiation behaves as if it is made up of particles called photons.

(ii) Every photon has energy $E = h\nu$ and momentum $p = \frac{h\nu}{c}$ and C is the speed of light.

(iii) All photons of light of particular frequency and wavelength have same energy and momentum irrespective of the intensity.

(iv) Photons are electrically neutral and they are not deflected by electric and magnetic fields.

(v) In a photon - particle collision, the total energy and total momentum are conserved. But the number of photons may not be conserved.

4. What is the de Broglie wavelength of a ball of mass 0.12 kg moving with a speed of 20 m/s? What can we infer from this result?

A. m = 0.12 kg; v = 20 m/s

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}}{0.12\times20} = 2.7625\times10^{-34}\,m$$

The de Broglie wavelength of a ball is about 10^{-19} times the size of proton.

Long Answer Questions

1. How did Einstein's Photoelectric Equation explain the effect of intensity and potential on photoelectric current? How did this equation account for the effect of frequency of incident radiation on stopping potential?

A. Einstein's Photoelectric Equation:

According to Einstein, when a photon strikes an electron, then its entire energy ($E = h\nu$) is utilized in two ways.

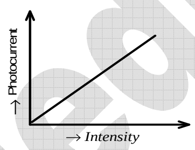
(i) A part of incident energy is used to just to eject an electron from the surface called the work function (ϕ_0) of the metal.

(ii) Remaining part of energy is converted as the kinetic energy of the ejected electron.

$$\therefore h\upsilon = \phi_0 + \frac{1}{2}mv^2$$

Effect of the Intensity:

For a given frequency of intensity of light, there is saturation photocurrent. This saturation photocurrent is proportional to the intensity of the light.

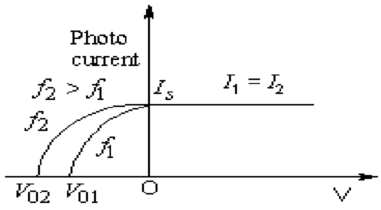


Intensity of light is proportional to the number of photons per unit area per unit time. Greater the number of photons, greater the photoelectrons (for $\nu > \nu_0$). Hence photocurrent is proportional to the intensity of light.

Effect of Potential:

For a given photo metal (ϕ_0 = constant), the kinetic energy of the ejected electron depends on the frequency of incident light. Greater the frequency, greater the kinetic energy. The value of the retarding potential difference to be applied between the surface of a photosensitive plate and the electrode of the collector, which is just sufficient to stop the most energetic photo electrons is called stopping potential. The stopping potential or cut off potential (V_S) is measure of the maximum K.E. of the emitted photoelectrons.

$$eV_S = K = \frac{1}{2}mv^2$$

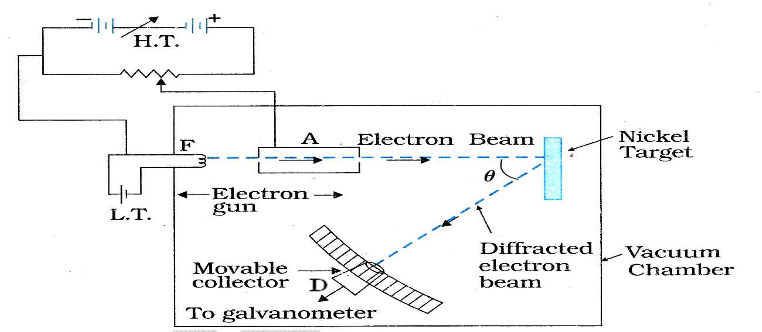


The value of stopping potential difference is independent of the light intensity. Stopping potential (V_S) is proportional to the frequency of incident radiation. Higher the frequency, larger the value of stopping potential, as shown in figure.

2. Describe Davisson and Germer experiment. What did this experiment conclusively prove?

A. Davisson and Germer Experiment:

The wave nature of electron was first explained by Davisson and Germer Experiment. They observed diffraction effects with beams of electrons scattered by crystals.



The experimental arrangement is shown in the figure .It consists of an electron gun with a tungsten filament 'F' coated with barium oxide and heated by a low voltage power supply. Electrons emitted by the filament are accelerated to a desired velocity by a suitable high voltage power supply. These are passed through a cylinder with fine holes along its axis, to produce a collimated beam. The electron beam is made to fall on the surface of nickel crystal and are scattered in all directions by the atoms of the crystal. The intensity of the electron beam scattered in a given direction is measured by the electron detector (collector).The detector can be moved on a circular scale and is connected to a sensitive galvanometer, which records the current. The deflection of the galvanometer is proportional to the intensity of the electron beam entering the collector. The apparatus is enclosed in an evacuated chamber.

By moving the detector on the circular scale at different positions, the intensity of the scattered electron beam is measured for different values of angle of scattering (θ) which is the angle between the incident and the scattered electron beams. The variation of intensity (I) of the scattered electron with the angle of scattering (θ) is obtained for different accelerating voltages.

For the range of voltages 44V to 68 V, it was noticed that a strong peak appeared in the intensity (I) of the scattered electron for an accelerating voltage of 54V at a scattering angle $\theta = 50°$. The appearance of the peak in a particular direction is due to the constructive interference of electrons scattered from different layers of the regularly spaced atoms of the crystals. The de-Broglie wavelength $'\lambda'$ associated with electrons,

$.\lambda = \frac{h}{p} = \frac{1.227}{\sqrt{V}} nm$ For V = 54 volt, $\lambda = 0.167 nm$

From the electron diffraction measurement, the wavelength of matter waves was found to be 0.165nm

PROBLEMS

1. Find the: (a) Maximum Frequency and (b) Minimum Wavelength of X-rays produced by 30kV electrons?

A. (a) $V_{max} = \frac{eV}{h}$

$$= \frac{1.6\times10^{-19}\times30\times10^{3}}{6.63\times10^{-34}}$$

$$= 7.24\times10^{18}\,Hz$$

(b) $$\lambda_{min} = \frac{c}{v_{max}} = \frac{3\times10^{8}}{7.24\times10^{18}}$$

$$= 0.0414\times10^{-9}\,m$$

= 0.0414 nm.

2. The work function of cesium metal is 2.14 eV. When light of frequency 6 x 10^{14} Hz is incident on the metal surface, photoemission of electrons occurs. What is the

(a) Maximum kinetic energy of the emitted electrons,

(b) Stopping potential and

(c) Maximum speed of the emitted photoelectrons?

A. Work function of cesium metal ϕ_0 = 2.14 eV

Frequency of light υ = 6 x 10^{14} Hz

(a) Maximum kinetic energy of emitted electrons (Einstein's photo electric equation)

$$KE_{max} = h\upsilon - \phi_0 = (6.63\times10^{-34}\times6\times10^{14})\text{ J - 2.14 ev}$$

$$= \left(\frac{6.63\times10^{-34}\times6\times10^{14}}{1.6\times10^{-19}}\right)eV - 2.14eV = 0.35\text{ eV}$$

(b) Let stopping potential be V_0.

But, $KE_{max} = eV_0$

$$0.35\text{ eV} = eV_0 \Rightarrow V_0 = 0.35\text{ V.}$$

(c) Maximum kinetic energy $KE_{max} = \frac{1}{2}mv^2_{max}$

$$0.35 \text{ eV} = \frac{1}{2}mv^2_{max}$$

(Where, v_{max} is the maximum speed and m is the mass of electron)

Or $$\frac{0.35\times2\times1.6\times10^{-19}}{9.1\times10^{-31}} = v^2_{max}$$

Or $$v_{max} = 344.8 \text{ km/sec}$$

3. The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?

A. $V_0 = 1.5V$

$$K_{max} = eV_o = 1.5eV$$

$$= 1.5\times1.6\times10^{-19} J$$

$$= 2.4\times10^{-19} J$$

4. Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42 mW.

(a) Find the energy and momentum of each photon in the light beam.

(b) How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross-section, which is less than the target area).

(c) How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon?

A. Wavelength of monochromatic light, $\lambda = 632.8$ nm $= 632.8 \times 10^{-9}$ m

Power = 9.42 mW = 9.42×10^{-3} W

(a) Energy of each photon, $E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{632.8\times10^{-9}} = 3.14 \times 10^{-19}$ J

We know that momentum of each photon, $p = \frac{h}{\lambda}$

$$p = \frac{6.63\times10^{-34}}{632.8\times10^{-9}} = 1.05 \text{ x } 10^{-17} \text{ kg-m/s}$$

(c) Momentum p = mv

Velocity of hydrogen atom, $v = \frac{p}{m} = \frac{1.05\times10^{-27}}{1.66\times10^{-27}} = 0.63$ m/s

[$\because$ m = 1.66 x 10^{-27} kg (mass of electron)]

(b) Let n be the number of photons per second. So,

$$n = \frac{\text{Power}}{\text{Energy of each photon}} = \frac{9.42\times10^{-3}}{3.14\times10^{-19}} = 3 \text{ x } 10^{16} \text{ photon/s}$$

5. The energy flux of sunlight reaching the surface of the earth is $1.388\times10^{3} Wm^{-2}$. How many photons (nearly) per square metre are incident on the Earth per second? Assume that the photons in the sunlight have an average wavelength of 550nm?

A. $P = \frac{nE}{t} \Rightarrow n = \frac{Pt}{E}$

$$\therefore n = \frac{1.388\times10^{3}\times550\times10^{-9}\times1}{6.63\times10^{-34}\times10^{8}} = 3.8\times10^{21}$$

6. In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be $4.12\times10^{-15} Vs.$ Calculate the value of Planck's constant.

A. $\frac{\Delta V}{\Delta v} = 4.12\times10^{-15} Vs,$

$e = 1.6\times10^{-19} C$

Planck's constant,

$h = \frac{\Delta V}{\Delta v}.e = 4.12\times10^{-15} 1.6\times10^{-19}$

$= 6.592\times10^{-34} Js.$

7. A 100 W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength of the sodium light is 589 nm?

(a) What is the energy per photon associated with the sodium light?

(b) At what rate are the photons delivered to the sphere?

A. Power of lamp, P = 100 W

Wavelength of the sodium light, $\lambda = 589$ nm = 589×10^{-9} m.

Planck constant h = 6.63×10^{-34} J-s.

(a) Energy of each photon

$$E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{589\times10^{-9}} \quad (c = 3 \times 10^{8} \text{ m/s})$$

$$= 3.38 \times 10^{-19} \text{ J} = \frac{3.38\times10^{-19}}{1.6\times10^{-19}} \text{eV} = 2.11 \text{ eV.}$$

(b) Let n photons are delivered per second

$$n = \frac{\text{Power}}{\text{Energy of each photon}} \quad \text{(From P = En)}$$

$$= \frac{100}{3.38\times10^{-19}} = 3 \times 10^{20} \text{ photon/s} = 3 \times 10^{20} \text{ photons/s are delivered.}$$

8. The threshold frequency a certain metal is $= 3.3\times10^{14}\,Hz$. If light of frequency $= 8.2\times10^{14}\,Hz$. is incident on the metal, predict the cut of voltage for photoelectric emission. Given $h = 6.63\times10^{-34}\,Js$ and $e = 1.6\times10^{-19}\,C$.

A. $v_0 = 3.3\times10^{-14}\,Hz, v = 8.2\times10^{14}\,Hz, V_0 = ?$

Maximum K.E. of a photoelectron is

$$eV_0 = hv - hv_0$$

$$\therefore\ V_0 = \frac{h(v-v_0)}{e}$$

$$= \frac{6.63\times10^{-34}\times(8.2-3.3)\times10^{14}}{1.6\times10^{-19}} \Rightarrow 2.03 \text{ V.}$$

9. The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?

A. Work function ϕ_0 = 4.2 eV = 4.2 x 1.6 x 10^{-19} J = 6.72 x 10^{-19} J

Wavelength of radiation, λ = 330 nm = 330 x 10^{-9} m

If the energy of each photon is more than the work function, then only the photoelectric emission takes place.

Energy of each photon

$$E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{330\times10^{-9}} = 6.027 \times 10^{-19} \text{ J.}$$

As the value of energy of each photon, E = 6.027 x 10^{-19} J is less than the work function, ϕ_0 = 6.72 x 10^{-19} J. So, no photoelectric emission takes place.

10. Light of frequency 7.21 x 10^{14} Hz is incident on a metal surface. Electrons with a maximum speed of 6.0 x 10^{5} m/s are ejected from the surface. What is the threshold frequency for photoemission of electrons?

A. Frequency of light, v = 7.21 x 10^{14} Hz

Mass of electron, m = 9.1 x 10^{-31} kg.

Maximum speed of electrons, v_{max} = 6 x 10^{5} m/s.

Let υ_0 be the threshold frequency.

Use the formula for kinetic energy

$$KE = \frac{1}{2}mv_{max}^2 = h\upsilon - h\upsilon_0$$

i.e., $\frac{1}{2}\times9.1\times10^{-31}\times6\times10^{5}\times6\times10^{5}$ = 6.63 x 10^{-34} $(\upsilon - \upsilon_0)$

Or $v - v_0 = \frac{36\times9.1\times10^{-21}}{2\times6.63\times10^{-34}}$ = 2.47 x 10^{14}

Or v_0 = 7.21 x 10^{14} - 2.47 x 10^{14} = 4.74 x 10^{14} Hz. (v = 7.21 x 10^{14} Hz)

11. Light of wavelength 488nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the cathode, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the cathode is made?

A. $\lambda = 488nm = 488\times10^{-9}m, V_0 = 0.38V$

From Einstein's photoelectric equation,

$$K_{\max} = \frac{1}{2}mv_{\max}^2 = h\nu - W_0$$

Or

$$eV_0 = \frac{hc}{\lambda} - W_0$$

$$\therefore\ W_0 = \frac{hc}{\lambda} - eV_0$$

$$= \frac{6.63\times10^{-34}\times3\times10^{8}}{488\times10^{-9}} - 1.6\times10^{-19}\times0.38$$

$$= 3.468\times10^{-19}J$$

Or

$$W_0 = 3.47\times10^{-19}J = 2.17eV.$$

12. Calculate the (a) Momentum and (b) de Broglie wavelength of the electrons accelerated through a potential difference of 56 V?

A. Potential difference, V = 56 V

(a) Use the formula for kinetic energy

$$eV = 1/2\ mv^2$$

$$2eV/m = v^2$$

$$v = \sqrt{\frac{2eV}{m}}$$

Where, m is mass and v is velocity of electron.

Momentum associated with accelerated electron,

$$p = mv = m\sqrt{\frac{2eV}{m}} = \sqrt{2eVm} = \sqrt{2\times1.6\times10^{-19}\times56\times9\times10^{-31}}$$

$= 4.02 \times 10^{-24}$ kg-m/s.

(b) de-Broglie wavelength of electron,

$$\lambda = \frac{12.27}{\sqrt{V}}\overset{\circ}{A} = \frac{12.27}{\sqrt{56}} = 0.164 \times 10^{-8}\ m = 0.164\ nm.$$

13. What is the (a) Momentum, (b) Speed and (c) de Broglie Wavelength of an electron with kinetic energy of 120 eV?

A. Kinetic energy = KE = 120 eV

(a) Momentum, $p = \sqrt{2eVm} = \sqrt{2KE.m}$ and $e = 1.6 \times 10^{-19}C$ $(\because KE = eV)$

$$= \sqrt{2\times120\times1.6\ 10^{-19}\times9.1\times10^{-31}} = 5.88 \times 10^{-24}\ \text{kg-m/s.}$$

(b) We know that momentum p = mv

or $v = p/m = \frac{5.91\times10^{-24}}{9.1\times10^{-31}} = 6.5 \times 10^{6}$ m/s.

(c) de-Broglie wavelength associated with electron,

$$\lambda = \frac{12.27}{\sqrt{V}}\overset{\circ}{A} = \frac{12.27}{\sqrt{120}}\overset{\circ}{A} = 0.112 \times 10^{-9}\ m = 0.112\ nm.$$

14. The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which (a) An Electron and (b) A neutron would have the same de-Broglie wavelength?

A. Wavelength of light 589 nm = 589×10^{-9} m

Mass of electron $m_e = 9.1 \times 10^{-31}$ kg.

Mass of neutron $m_n = 1.67 \times 10^{-27}$ kg

Planck's constant $h = 6.62 \times 10^{-34}$ J-s.

(a) Using of formula, $\lambda = \frac{h}{\sqrt{2KEm_e}}$

Kinetic energy of electron,

$$KE_e = \frac{h^2}{2\lambda^2 m_e} = \frac{(6.63\times10^{-34})^2}{2\times(589\times10^{-9})^2\times9.1\times10^{-11}} = 7.03 \text{ x } 10^{-25} \text{ J.}$$

(b) Kinetic energy of neutron

$$KE_n = \frac{h^2}{2\lambda^2 m_n} = \frac{(6.63\times10^{-34})^2}{2\times(589\times10^{-9})^2\times1.66\times10^{-27}} = 3.81 \text{ x } 10^{-28} \text{ J.}$$

15. What is the de-Broglie wavelength of

(a) A bullet of mass 0.040 kg travelling at the speed of 1.0 km/s

(b) A ball of mass 0.060 kg moving at a speed of 1.0 m/s and

(c) A dust particle of mass 1.0 x 10^{-9} kg drifting with a speed of 2.2 m/s?

A. Mass of bullet m = 0.040 kg and speed of bullet v = 1000 m/s

(a) de-Broglie wavelength

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}}{0.040\times1\times10^{3}} \quad \left(\begin{array}{l} \because m = 0.04\,kg \\ v = 1\,km/s \\ \quad = 1000\,m/s \end{array}\right)$$

$= 1.66 \text{ x } 10^{-35}$ m.

(b) Mass of the ball, m = 0.60 kg and speed of the ball, v = 1 m/s

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}}{0.060\times1} = 1.1 \text{ x } 10^{-32} \text{ m}$$

(c) Mass of a dust particle, m = 1 10^{-9} kg and speed of the dust particle, v = 2.2 m/s

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}}{1\times10^{-9}\times2.2} = 3.0 \text{ x } 10^{-25} \text{ m.}$$

16. An electron and a photon, each have a wavelength of 1.00 nm. Find

(a) Their momenta,

(b) The energy of the photon and

(c) The kinetic energy of electron.

A. Wavelength of electron and photon, λ = 1 nm = 10^{-9} m

(a) Momentum of electron,

$$p_e = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{10^{-9}} = 6.63 \times 10^{-25} \text{ m } (h = 6.63 \times 10^{-34} \text{ Js})$$

Momentum of photon,

$$p_{ph} = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{10^{-9}} = 6.63 \times 10^{-25} \text{ m.}$$

(b) Energy of photon,

$$E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{10^{-9}\times1.6\times10^{-19}} eV$$

$$= \frac{19.86\times10^{-17}}{1.6\times10^{-19}} eV = 1243 \text{ eV or } E = 1.24 \text{ keV.}$$

(c) Energy of electron,

$$E = \frac{p^2}{2me} = \frac{(6.63\times10^{-25})^2}{2\times9.1\times10^{-31}\times1.6\times10^{-19}} eV = 1.51 \text{ eV.}$$

17. (a) For what kinetic energy of a neutron will the associated de-Broglie wavelength be

(b) Also find the de-Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of (3/2) kT at 300 K.

A. **(a)** de-Broglie wavelength $\lambda = 1.40\times10^{-10}$ m

Mass of neutron, $m_n = 1.675 \times 10^{-27}$ kg

Using the formula, wavelength associated with kinetic energy

$$\lambda = \frac{h}{\sqrt{2m\,KE}}$$

Or $$KE = \frac{h^2}{2\lambda^2 m_n} = \frac{(6.63\times10^{-34})^2}{2\times(1.40\times10^{-10})^2\times1.675\times10^{-27}}$$

$= 6.686 \times 10^{-21}$ J.

(b) Kinetic energy associated with temperature

$$KE = \frac{3}{2}kT = \frac{3}{2}(1.38\times10^{-23})\times300 = 6.21 \times 10^{-21} \text{ J.}$$

($\because$ Absolute temperature T = 300 K and Boltzmann's constant k = 1.38×10^{-23} J/K)

KE = 6.21×10^{-21} J

De-Broglie wavelength associated with kinetic energy

$$\lambda = \frac{h}{\sqrt{2m\,KE}}$$

$$\lambda = \frac{6.63\times10^{-34}}{\sqrt{2\times1.675\times10^{-27}\times6.21\times10^{-21}}} = 1.45 \times 10^{-10} \text{ m} = 1.45$$

18. Show that the wavelength of electromagnetic radiation is equal to the de Broglie wavelength of its quantum (photon)?

A. For a photon,

De Broglie wavelength, $\lambda = \frac{h}{p}\cdot$

For an electromagnetic radiation of frequency v and wavelength $\lambda'(= c/v)$,

Momentum, $P = \frac{E}{c} = \frac{hv}{c}$

Or

$$P = \frac{h}{c}\cdot\frac{c}{\lambda}\cdot = \frac{h}{\lambda'}$$

Then, $\lambda' = \frac{h}{p} = \lambda$

Thus the wavelength λ' of the electromagnetic radiation is the same as the de-Broglie wavelength λ of the photon.

19. What is the de-Broglie wavelength of a nitrogen molecule in air at 300K? Assume that the molecule is moving with the root-mean-square speed of molecules at this temperature? (Atomic mass of nitrogen $= 14.0076v, k = 1.38\times10^{-23} JK^{-1}$)

A. Mass of N_2 molecule,

$$m = 2\times14.0076\times1.66\times10^{-27} kg$$

$$= 46.5\times10^{-27} kg$$

T = 300 K

$$\therefore \lambda = \frac{h}{\sqrt{3mKT}}$$

$$= \frac{6.63\times10^{-34}}{\sqrt{3\times46.5\times10^{-27}1.38\times10^{-23}\times300}} m$$

$$= 0.0276\times10^{-9} m = 0.028 nm.$$