NUCLEI

Important Points:

1. The nuclei having the same atomic number (Z), but different mass numbers (A) are called **isotopes**.

Ex: ${}_{1}^{1}H$, ${}_{1}^{2}H$, ${}_{1}^{3}H$ are the isotopes of hydrogen atom.

2. The nuclei having the same neutron number (N) but different atomic numbers (Z) are called **isotones**.

Ex: ${}_{80}Hg^{198}$, ${}_{79}Au^{197}$, ${}_{20}Ca^{40}$, ${}_{19}K^{39}$

3. The nuclei having the same mass number (A), but different atomic numbers (Z) are called **isobars.**

Ex: ${}_{6}C^{14}$, ${}_{7}N^{14}$, ${}_{32}Ge^{76}$, ${}_{34}Se^{76}$

4. Nuclei having the same atomic number (Z) and mass number (A) but with different nuclear properties such as radioactive decay and magnetic moments are called isomers.

Ex: ${}_{35}Br^{80}m$, ${}_{35}Br^{80}g$

Here m denotes meta-stable state and 'g' denotes ground state.

5. The volume of the nucleus (V) is found to be proportional to its mass number (A).

$R = R_0 A^{1/3}$, Where R_0 is constant of proportionality

6. **Mass Defect and Binding Energy:**

a) The difference between the total mass of all the nucleons of the nucleus and the actual mass of a nucleus is called "mass defect", It is denoted by Δm .

b) Packing fraction: The mass defect per nucleon is called packing fraction or the ratio of mass defect to the mass number is called packing fraction. $pf = \frac{\Delta m}{A}$.

7. **Relation between Mass Defect and Binding Energy:**

Mass defect (Δm) = (Total mass of the protons + Total mass of the neutrons) - (Actual mass of the nucleus)

$\Delta m = \left[Zm_p + (A - Z) m_n - m_N \right]$ Where, m_e= mass of electron

8. The binding energy can be defined as the energy released when protons and neutrons combine to form a nucleus.

Binding energy $(\Delta E) = \Delta mc^2$

9. Natural Radio Activity:

The nuclei of certain elements disintegrate spontaneously by emitting alpha (α), beta(β), and gamma(γ) rays. This phenomenon is called natural radio activity.

10. Alpha Radiation:

When a nucleus emits an α-particle its atomic number (Z) decreases by two units and its mass number (A) decreases by four units. ${}_{92}U^{238} \rightarrow {}_{90}Th^{234} + {}_{2}He^{4}$

11. Beta Radiation:

a) When a nucleus emits a β-particle, the atomic number of the nucleus increases by one unit, but the mass number does not change. ${}_{90}Th^{234} \rightarrow {}_{91}Pa^{234} + {}_{-1}\beta^{0}$

b) Both electric charge and nucleon number are conserved in –decay

12. Gamma Radiation:

a) γ-rays are nothing but electromagnetic radiations of short wave lengths (not exceeding 10^{-10}m).

b) The emission of γ-rays from the nucleus does not alter either atomic number (Z) (or) mass number (A).

13. Radio - Active Decay Law:

a) The number of nuclei decaying per unit time at any instant $\left(\frac{dN}{dt}\right)$ is directly proportional to the number of nuclei (N) present at that instant.

$$\frac{dN}{dt} \alpha - N \quad \text{Or} \quad N = N_0 e^{-\lambda t}$$

b) Activity of a radioactive sample = λN

c) The S.I unit for activity of radioactive substance is Becquerel (Bq)

14. Half Life of a Radio Active Substance:

It the time required for the number of radioactive nuclei of the substance disintegrate to half of its original number of nuclei.

$$T = \frac{0.693}{\lambda}$$

Half-life (T) depends upon the disintegration constant (λ) of the radioactive substance.

15. Average Life of a Radio Active Substance:

Average life time (τ) is equal to the total life time of all the nuclei divided by the total number of original nuclei N_0. $\tau = \frac{1}{\lambda}$

16. Nuclear Fission:

The phenomenon of splitting of a heavy nucleus (usually A > 230) into two or more lighter nuclei is called Nuclear fission.

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3{}^{1}_{0}n + Q$$

21. Nuclear Fusion:

The proton - proton cycle is a source of energy in the sun and other stars of comparatively lower temperatures (Red-dwarfs)

$$4{}^{1}_{1}H \rightarrow {}^{4}_{2}He + 2{}^{0}_{+1}e + 25.71\text{MeV} \text{ Of energy}$$

Very Short Answer Questions

1. What are Isotopes and Isobars?

A. **Isotopes:**

Nuclei having same atomic number (Z), but different mass numbers (N) are called isotopes.

Ex: 1) ${}_{1}^{1}H$, ${}_{1}^{2}H$, ${}_{1}^{3}H$ are the isotopes of hydrogen.

2) ${}_{8}^{16}O$, ${}_{8}^{17}O$, ${}_{8}^{18}O$ are the isotopes of oxygen.

Isobars:

Nuclides having the same mass number (A), but different atomic numbers (Z) are called isobars.

Ex: 1) ${}_{6}^{14}C$, ${}_{7}^{14}N$, 2) ${}_{32}^{76}Ge$, ${}_{34}^{76}Se$

2. What are Isotones and Isomers?

A. **Isotones: -** Nuclei having the same neutron number (N) but different atomic numbers (Z) are called isotones.

Ex: ${}_{20}^{40}Ca$, ${}_{19}^{39}K$ 2) ${}_{80}^{198}Hg$, ${}_{79}^{197}Au$

Isomers:-Nuclides having same atomic number (Z) and mass number (A) but with different nuclear properties such as radioactive decay and magnetic moment, etc... are called **Isomers**.

Ex: ${}_{35}^{80}Br^{m}$, ${}_{35}^{80}Br^{g}$ Here m = meta stable state

g = ground state

3. What is amu? What is its equivalent energy?

A. Atomic Mass Unit (A.M.U):

It is defined as $\frac{1}{12}th$ of the mass of the carbon $\left({}^{12}C\right)$ atom.

1 amu (u) = 931.5 Mev

4. What will be the ratio of the radii of two nuclei of mass numbers A_1 and A_2?

A. A Nucleus of mass number 'A' has a radius, $R = R_0A^{1/3}$. Ratio of radii of two nuclei of mass numbers A_1 and A_2 is

$$\frac{R_1}{R_2} = \frac{R_0A_1^{1/3}}{R_0A_2^{1/3}} = \left(\frac{A_1}{A_2}\right)^{1/3}.$$

5. Natural radioactive nuclei are mostly nuclei of high mass number why?

A. In all naturally occurring heavy radioactive nuclides, n/p value is more than 1.56. These are more unstable. So they undergo radioactive decay.

6. Does the ratio of neutrons to protons in a nucleus increase, decrease or remain the same after the emission of an α - particle?

A. After the emission of an α - particle, the ratio of neutron to proton is increases.

7. A nucleus contains no electrons but can emit them. How?

A. When a nucleus disintegrates and radiates β-rays, it undergoes β–decay. β-particle is nothing but fast moving electron. In the conversion of a neutron into a proton a β- particle is emitted.

$$n \to p + e^- + \bar{v}$$

8. What are the units and dimensions of the disintegration constant?

A. SI unit: $(\text{sec})^{-1}$

Dimensional formula: $[M^0L^0T^{-1}]$

9. Why do all electrons emitted during β- decay not have the same energy?

A. The energy released in β- decay is shared by the electrons and anti neutrino ($\bar{v}$). Hence different electrons possess different energies.

10. Neutrons are the best projectiles to produce nuclear reactions. Why?

A. Neutrons are electrically neutral and they are not deflected by both magnetic and electric field. They have high penetrating power. Therefore neutrons enter the nucleus easily and are not deflected by the positive charge of the nucleus. Thus neutrons are the best projectiles to produce nuclear reactions.

11. Neutrons cannot produce ionization. Why?

A. As neutrons are electrically neutral and highly penetrating, they cannot produce ionization.

12. What are delayed neutrons? What is its importance?

A. Delayed Neutrons:

In nuclear fission process, few neutrons (around 1%) are emitted over a period of time. These neutrons are called delayed neutrons.

Importance:

These neutrons play an important role in the working of nuclear reactor by accelerating the fission process.

13. What are thermal neutrons? What is their importance?

A. Thermal Neutrons:

If fast moving neutrons pass through substances like heavy water, paraffin wax, graphite etc., they are slowed down to thermal energy levels. These neutrons are called thermal neutrons.

Importance:

These are used in nuclear fission reactions.

14. What is the value of neutron multiplication factor in a controlled reaction and in an uncontrolled chain reaction?

A. Neutron multiplication factor K = No. of neutrons in the present generation / No. of neutrons in the previous generation.

For controlled chain reaction, $K = 1$

For uncontrolled chain reaction, $K > 1$

15. What is the role of controlling rods in a nuclear reactor?

A. **C**ontrol rods are the neutron absorbing materials like cadmium, boron etc. These absorb the neutrons and there by controls the nuclear fission process.

16. Why are nuclear fusion reactions called Thermo Nuclear Reactions?

A. Nuclear fusion reactions occurs only at very high temperatures of the order of 10^7K. These are also known as thermo nuclear reactions.

17. Define Becquerel and Curie?

A. The S.I unit of activity is Becquerel (Bq) which is equal to 1 decay or disintegration per second.

1 Bq = 1 decay per second

Curie:

The activity of a radioactive substance one Curie (Ci) which is equal to 3.7×10^{10} decays or disintegrations per second.

1 Ci = 3.7×10^{10}

18. What is a chain Reaction?

A. A chain reaction is a self propagating process in which a number of neutrons multiply rapidly during fission till the whole fissionable material is disintegrated.

19. What is the function of moderator in a nuclear reactor?

A. The purpose of the moderator is to slow down the fast moving neutrons produced as a result of nuclear fission.

Ex: Heavy water, beryllium, carbon in the form of pure graphite, hydrocarbon plastics etc.

20. What is the energy released in the fusion of four protons to form a helium nucleus?

A. $4\,{}_1^1H \rightarrow {}_2^4He + 2\,{}_1^0e + 26.7MeV$

Energy released = 26.7 Mev.

Short Answer Questions

1. Why is the density of the nucleus more than that of the atom? Show that the density of nuclear matter is same for all the nuclei?

A. Most part of the atom is hollow. The entire mass is concentrated at the centre of atom i.e. nucleus. Thus, mass per unit volume (density) of the nucleus is more than that of the atom.

Density of the Nucleus:

Volume of the nucleus = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi (r_0 A^{1/3})^3 = \frac{4}{3}\pi r_0^3 A$

Mass of each proton = $1.670020\times10^{-27}\,kg$

Mass of the nucleus = $1.670020\times10^{-13}\,A$

$$\text{Density} = \frac{mass}{volume} = \frac{1.67\times10^{-27}\times A}{\frac{4}{3}\pi r_0 \times A} \approx 10^{17}\,kg/m^3$$

Density of the nucleus does not depend on mass number (A) .Hence the density of nuclear matter is same for all nuclei.

2. Write a short note on the discovery of neutron?

A. Discovery of Neutron:

Bothe and Becker observed that a highly penetrating radiation was emitted when Boron or Beryllium were bombarded with particles of energy about 5 MeV. These were thought to be high energetic -rays because these are not affected by electric or magnetic fields.

$$_4Be^9 + {}_2He^4 \rightarrow \left[{}_6C^{13}\right] \rightarrow {}_6C^{13} + \gamma$$

Absorption measurements estimated that these -photon energy should be about 7MeV. Later Curie and Joliot observed that when this radiation is passed through hydrogenated materials like paraffin etc. A high energy protons were ejected with a maximum energy of about 5MeV. From the calculation, it has shown that ejection of 5MeV protons require γ-photons of energy 55MeV. Thus this lea to controversies about the energy of the γ-photons.

Later in the year 1932 Chadwick concluded that these are a group of neutral particles of mass equal to that of protons. These neutral particles are called neutrons.

$$_4Be^9 + {}_2He^4 \rightarrow \left[{}_6C^{13}\right] \rightarrow {}_6C^{12} + {}_0n^1 + Q \qquad (\text{ Q = energy })$$

3. What are the properties of a neutron?

A. 1) Mass of the neutron is 1.00866 amu = 1.6749 x 10^{-27} Kg.

2) It is an uncharged particle. Hence it is not deflected by electric and magnetic fields.

3) These are having high penetrating power and low ionizing power.

4) These are more stable inside the nucleus and unstable outside the nucleus.

5) A free neutron decays into a proton, an electron (β - particle) and an antineutrino $(\bar{\nu})$.

$$n \to p + e^{-} + \bar{\nu}$$

6) Its Mean life is nearly 1000s.

7) These are fermions and having half -integral spins.

8) For non- relativistic velocities (V <<C), the wavelength associated with neutrons is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{0.286}{\sqrt{E}} \text{ A}^0 \text{ (Here 'E' is in eV)}$$

4. What are nuclear forces? Write their properties?

A. Nuclear Forces:

The force that holds the nucleons together in the nucleus is an attractive force and it is called nuclear force.

Properties:

1. Nuclear force is the strongest force of all the basic forces.
2. Nuclear forces are the short range force.
3. Nuclear forces are more effective if the separation between the nucleons if of order of 1 Fermi or less.
4. If the distance between the nucleons is less 0.4 Fermi, the nuclear forces become repulsive.
5. These are charge independent.
6. The magnitude of these forces between two protons or two neutrons or between a proton and neutron are same.
7. Nuclear forces are spin dependent.

8. Nuclear forces have saturation property, i.e., each nucleon interacts with its immediate neighbor only.

5. For greater stability, a nucleus should have greater value of binding energy per nucleon. Why?

A. Binding Energy per nucleon is the average energy per nucleon needed to separate a nucleus into its individual nucleons.

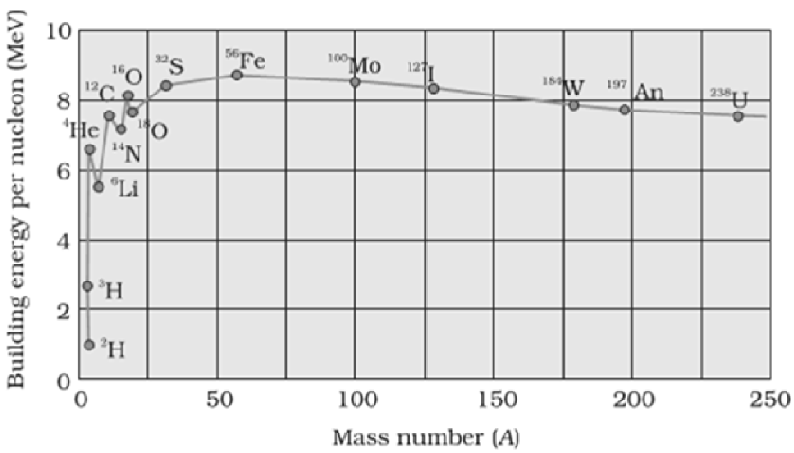
It is denoted by E_{bn}

$$\text{Average binding energy} = \frac{\text{Binding energy}}{\text{Atomic mass number}}$$

$$E_{bn} = \frac{E_b}{A}$$

E_{bn} is an indication of the stability of the nucleus.

If E_{bn} is more for nuclei, it is considered as more stable.



A graph is plotted for the Binding Energy per nucleon E_{bn} versus the mass number A for large number of nuclei as shown in fig. the main features of the graph are

(i) The Binding Energy per nucleon E_{bn} is practically constant i.e., practically independent of the atomic number for nuclei of middle mass number (30<A<170).

(ii) The curve has a maximum of about 8.75 Mev for Iron (A=56) Hence it is more stable and has a value of 7.6 Mev for Uranium (A=238) hence it is unstable.

(iii) E_{bn} is lower for both light nuclei (A<30) and heavy nuclei (A>170)

(iv) To attain greater stability, Uranium (A = 240) breaks up into intermediate mass nuclei resulting in a phenomenon called fission.

(v) Two lighter nuclei are combined to form a stable nucleus. This phenomenon is called fusion.

6. Explain α - decay.

A. **α - decay:**

(i) When a nucleus disintegrates by radiating α - rays, it is said to undergo α -decay.

(ii) When a nucleus emits an alpha particle its atomic number (Z) decreases by two units and its mass number a decreases by four units. The resultant nucleus corresponds to a different element.

(iii) The original nucleus is called parent nucleus and the resultant nucleus after disintegration is called daughter nucleus.

(iv) The general form of α -decay can be written as

$$ {}_Z^AX \rightarrow {}_{Z-2}^{A-4}Y + {}_2^4He $$

(v) Q - Value of above nuclear reaction is the difference between the initial mass energy and the total mass energy of the decay products.

i.e. $Q = [m_X - m_Y - m_{He}]C^2$.

Ex: ${}_{92}^{238}U \rightarrow {}_{90}^{234}Th + {}_2^4He$ (α -dcay)

7. Explain β – decay.

A. β – decay:

In the decay, a nucleus spontaneously electron $\left(\beta^- decay\right)$ or a position (β^+ decay)

For decay ${}_{15}^{32}P \rightarrow S + e^- + \bar{v}$

For decay ${}_{11}^{32}Na \rightarrow {}_{10}^{22}Ne + e^+ + v$

In β^- decay an anti neutrino $\left(\bar{v}\right)$ and in β^+ decay neutrino $\left(\bar{v}\right)$ are generated

In both β^- and β^+ decay, the mass number A remains unchanged. In β^- decay, the atomic number Z of the nucleus increases by 1, while in β^+ decay Z decreases by 1. The basic nuclear process underlying β^- decay is the conversion of neutron to proton.

$$n \to p + e^- + \bar{v}$$

While for β^+ decay, it is the conversion of proton into neutron

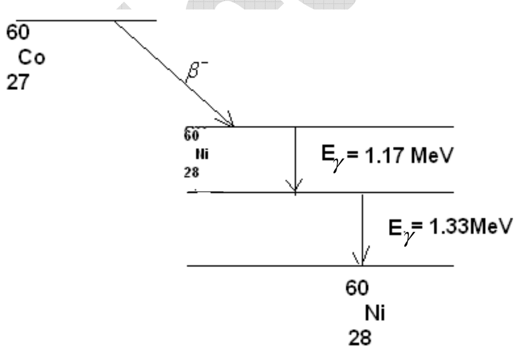
$$p \to n + e^+ + v$$

8. Explain γ - decay.

A. γ **- decay:**

When a nucleus disintegrates by radiating γ-rays, it is said to undergo γ-decay. Gamma rays are electromagnetic radiations having short wavelengths. Due to the emission of γ-rays, does not alter either atomic number (Z) (or) mass number (A). But change of the energy state of a nucleus is possible.

Ex: When $^{60}_{27}\mathrm{Co}$ emits β- particle, then the daughter nucleus ($^{60}_{28}\mathrm{N}i$) is left in the excited state. It suddenly makes transition from excited state to ground state there by emitting γ-rays as shown in fig.



Here charge and mass (nucleon) number are remains conserved.

9. Define half life period and decay constant for a radioactive substance. Deduce the relation between them?

A. Decay Constant:

Decay constant is defined as the ratio of its instant rate of disintegration to the number of atoms present at that time.

$$\lambda = \frac{dN/dt}{N}$$

Half life $(T_{1/2})$:

Time interval in which the mass of a radioactive substance or the number of its atom reduces to half of its initial value is called the half life of the substance.

i.e. if $N = \frac{N_0}{2}$

Then $t = T_{1/2}$

Hence from $N = N_0 e^{-\lambda t}$

$$\frac{N_0}{2} = N_0 e^{-\lambda(T_{1/2})} \Rightarrow T_{1/2} \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda}$$

10. Define average life of a radioactive substance. Obtain the relation between decay constant and average life?

A. **Average life time (τ):** The average life time is defined as the ratio of the total life time of all the N_0 nuclei to the total number of original nuclei N_0.

Average life (τ) = $\int \frac{tdN}{N_0}$ ……. (1)

But $\frac{dN}{dt} = -\lambda N$

dN = -N dt (∵ $N = N_0 e^{-\lambda t}$)

$dN = -N_0 e^{-\lambda t} \lambda\, dt$

To obtain mean life, integrate equation (1) from $0 \to \infty$

$$\tau = \frac{\int_0^{\infty} t N_0 e^{-\lambda t} \lambda dt}{N_0}$$

On integration we get $\tau = \frac{1}{\lambda}$

The reciprocal of the decay constant is the average life of a radioactive substance.

11. Deduce the relation between half life and average life of a radioactive substance?

A. Half-Life:

The half life of a radioactive substance is defined as the time during which half of the atoms of the substance will disintegrate.

The relation between half-life (T) and disintegration constant (λ) of radioactive substance is

$$T = \frac{0.693}{\lambda} \text{...........(1)}$$

Mean life time (or) average life time (T_{avg}):

It is the ratio of total life of all the atoms of a given sample to the total number of atoms present in the sample.

$$T_{avg} = \left(\frac{1}{\lambda}\right) \text{...........(2)}$$

From (1) and (2) we have relation between average life and half-life of a radioactive substance as

$$T = 0.693 \Rightarrow T_{avg} = \frac{T}{0.693}$$

12. What is nuclear fission? Give an example to illustrate it?

A. **Nuclear Fission:**

The phenomenon in which a heavy unstable nucleus breaks into two fragments of nearly same mass is called nuclear fission.

Ex: Consider ${}^{235}_{92}U$ nucleus was bombarded with slow neutrons. It splits into Barium and krypton and the energy released is 200 MeV.

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3{}^{1}_{0}n + 200 \text{ MeV}$$

Mass defect Δm = total mass of reactants - Total mass of products.

Energy (E) = $\Delta m \times C^2 = \Delta m \times 931.5$ MeV

The binding energy per nucleon is 0.85 MeV

Hence energy released is 235× 0.85 MeV

E = 199.75 MeV

13. What is nuclear fusion? Write the conditions for nuclear fusion to occur?

A. Nuclear Fusion:

The process of the formation of a single stable nucleus by fusing two or more lighter nuclei is called nuclear fusion. The energy released in this process is called fusion energy.

Ex:- Formation of a helium nucleus by fusing of four hydrogen nuclei and releasing 26.7MeV of energy.

$$4\,{}_{1}^{1}H \rightarrow {}_{2}^{4}He + 2\,{}_{+1}^{0}e + Energy$$

In the above reaction the mass of the helium nucleus is smaller than the sum of the masses of four protons. This difference in mass appears as liberation of energy in the fusion process. In the fusion of four protons, the energy released per nucleon is about 6.5MeV.

Conditions:

To carry out the fusion of two nuclei, they must be brought very close to each other so that they overcome the repulsive force. This is only possible at a temperature nearly equal to 10^7 K, Such a high temperature is possible when an atomic bomb is explodes. Once the fusion takes place, the energy released can maintain the minimum required temperature for further fusion.

As fusion can occur at very high temperatures the fusion reactions are also known as thermo nuclear reactions.

14. Distinguish between nuclear fission and nuclear fusion.

A.

Nuclear Fission	Nuclear Fusion
1) This is the process of splitting of a heavier into two or more stable fragments.	1) This is the process of fusing two lighter nuclei into a heavier nucleus to attain stability.
2) Each fission gives about 200 MeV of energy.	2) Each fusion gives about 28 MeV of energy.

3) Energy released per nucleon is less and equal to 0.85 MeV.	3) Energy released per nucleon is more and equal to 6 MeV.
4) This is the principle of atom bomb.	4) This is the principle of hydrogen bomb.
5) Fission takes place at room temperature	5) Fusion takes place at high temperatures.
6) Energy is produced by nuclear reactors is by fission.	6) Energy released by stars and sun is by fusion.

15. Explain the terms 'Chain reaction' and multiplication factor. How is a chain reaction sustained?

A. Chain Reaction:

In a nuclear fission reaction three neutrons are released. Each of these neutrons inturn causes further fission in three more Uranium nuclei resulting in release of nine neutrons. These neutrons split nine more nuclei and release 27 neutrons. As this process continues the number of neutrons released increases in geometric progression and this process is called chain reaction.

Neutron Multiplication Factor:

The ratio of second generation neutrons to the first generation neutrons is called neutron multiplication factor.

Neutron multiplication factor (K) = $\frac{\text{No of neutrons is second generation}}{\text{No of neutrons in the first generation}}$

a) If K < 1, the reaction is not self sustained. It is called sub- critical state.

b) If K = 1, then it is called a controlled chain reaction called critical state.

c) If K>>1, the reaction is self sustained. It is called super critical state (principle of atom bomb).

Condition for Sustained Chain Reaction:

To sustain the chain reaction the mass of Uranium should be equal to or more than a particular amount of mass called 'critical masses**.**

Long Answer Questions

1. **Define mass defect and binding energy. How does binding energy per nucleon vary with mass number? What is its significance?**

A. **Binding Energy:**

The minimum amount of energy required to split the nucleus into its constituent nucleons is called binding energy.

Mass Defect:

The difference between the actual mass of a nucleus and the sum of the masses of the nucleons present in it is known as the “mass defect” (Δm).

$$\Delta m = [Zm_p + (A-Z)m_n] - M$$

Where m_p and m_n are the masses of proton and neutron respectively and M is the actual mass of the nucleus.

Z = Number of protons in the nucleus

A – Z = Number of neutrons

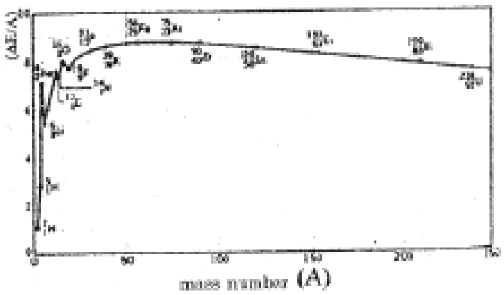
The energy equivalent of the mass defect is the binding energy of the nucleus.

Binding energy of the nucleus= Δm x 931.5 MeV.

Binding Energy Curve:

The ratio of binding energy of nucleus (E_b) and the total number of nucleons (A) the nucleus is called the binding energy per nucleon or binding fraction.

Binding fraction $(E_{bn}) = \dfrac{Binding\ energy\ of\ the\ nucleus}{A} = \dfrac{\Delta mc^2}{A}$



Binding energy per nucleon (ΔE/A) versus the Mass number (A).

The graph of binding energy per nucleon as a function of mass number is shown in the figure.

1) The binding energy per nucleon rises sharply to a maximum of about 8.8MeV in the neighborhood of A = 56, attains a 8.4MeV at about A = 140 and decreases to 7.6MeV for uranium.

2) The value of BE per nucleon is close to the maximum value for the elements of range 28<A<138.

3) In the region of smaller mass numbers ${}_{2}^{4}He$, ${}_{6}^{12}C$, and ${}_{8}^{16}O$, have maximum BE than those elements ${}_{3}^{6}Li$, ${}_{5}^{10}B$, and ${}_{7}^{14}N$, which are nearby. This shows that the nuclei having equal number of protons and neutrons are stable. The nuclei having even number of protons are more stable than those of odd number of protons.

Significance:

1) More the binding energy, more the stability.

2) A large amount of energy can be liberated if heavier nuclei can be split into lighter nuclei i.e. nuclear fusion.

3) A large amount of energy can be liberated if lighter nuclei can be made to fuse to form heavier nuclei i.e. nuclear fusion.

2. What is radioactivity? State the law of radioactive decay. Show that radioactive decay is exponential in nature?

A. Radioactivity:

The process of spontaneous disintegration of the nuclei of heavy elements with the emission of certain radiations is known as natural radioactivity.

The nuclei of certain elements with atomic number more than 82 disintegrate by emitting alpha (α), beta (β) and gamma (γ) rays. This phenomenon is called radioactivity.

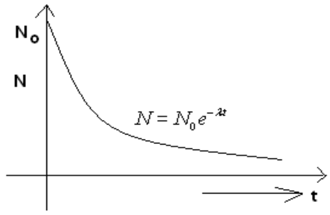
Radioactive Decay Law: The rate of disintegration, $\left(\frac{dN}{dt}\right)$ i.e. the number of atoms disintegrated per second is directly proportional to the number of atoms present (N) at that moment. This is known as radioactive decay Law.

Explanation:

Let N_0 be the number of atoms present in a radioactive sample initially (t = 0).

Let N be the number of atoms left at a time t and dN be the no of atoms disintegrating in a short interval of time t.

The rate of disintegration = $\frac{dN}{dt}$



According to decay law,

$$\frac{dN}{dt} \propto N \qquad \text{Or} \qquad \frac{dN}{dt} \propto -\lambda N$$

Where λ = decay or disintegration constant. The negative sign indicates that as time increases, the number of atoms decreases.

$$\text{Or} \qquad \frac{dN}{N} \propto -\lambda N$$

Integrating the above equation

$$\int \frac{dN}{N} = -\lambda \int dt \qquad \text{Or} \quad \log_e N = -\lambda t + c \ldots\ldots (1)$$

Where c = integration constant and it can be found in the initial condition.

i.e., If t = 0, N = N_0

Then, $\log_e N_0 = c$ ………… (2)

From equation (1)

$\mathrm{Log}_e N = -\lambda t + \log_e N_0$

$\mathrm{Log}_e N - \log_e N_0 = -\lambda t$ (or)

$$\log_e \left(\frac{N}{N_0}\right) = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t} \qquad \text{OR} \qquad N = N_0 e^{-\lambda t}$$

Hence the number of radioactive nuclei decreases exponentially with time and reduced to zero after infinite time.

3. Explain the principle and working of a nuclear reactor with the help of a labeled diagram.

A. Nuclear Reactor:

Nuclear reactor is used to produce a large amount of nuclear energy through a controlled nuclear fission process.

i) Nuclear Fuel:

The fissionable material used in the reactor is called nuclear fuel. Commonly used fuels in the reactors. The commonly used nuclear fuels in the reactors are Uranium isotopes ${}_{92}U^{235}$ and ${}_{92}U^{238}$ Plutonium Pu and thorium ${}_{99}th^{232}$ are.

ii) Moderators:

These are used to slow down the fast moving neutrons produced in the fission process. The materials used as moderators are heavy water, carbon in the form of pure graphite, hydrocarbon plastics etc. The core is surrounded reflector to reduce leakage.

iii) Control Rods:

These are the materials that can absorb the neutrons and control the nuclear chain reaction. Cadmium or Beryllium rods are generally used for this purpose.

iv) Safety Rods:

These are used to reduce the neutron reproduction rate to less than one abruptly to stop the chain reaction whenever required.

v) Protective Shielding:

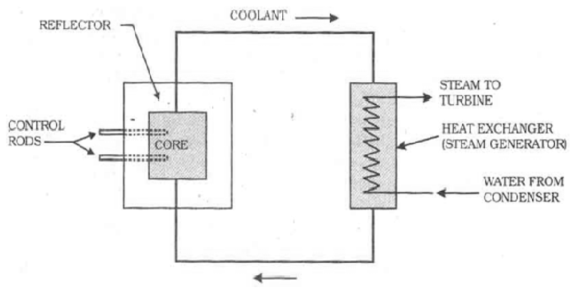
It is used to prevent the spreading of the radioactive effect to the space around the nuclear reactor. For this purpose lead blocks, concrete walls of thickness 10m are used.

vi)Coolant:

The material used to absorb the heat generated in the reactor is called coolant. The coolants are water, molten sodium etc.

Working:

Uranium fuels are placed in the aluminum cylinders which are separated by some distance. The graphite moderator in the form of pure carbon blocks is placed in between the fuel cylinders. To control the number of neutrons, a number of control rods of cadmium or beryllium or boron are placed in the holes of graphite block.



When a ^{235}U nuclei undergo fission fast neutrons are liberated. These neutrons pass through the surrounding graphite moderation and lose their kinetic energy to become thermal neutrons. These thermal neutrons are captured by ^{235}U which carries out the fission reaction.

By using the control rods the fission process can be controlled. The heat generated in this process is used for heating coolant which in turn heat water and produce steam. This steam is used to rotate a turbine for the production of electric power.

Uses of a Nuclear Reactor:

1) To generate electric power.

2) To produce radioactive materials like Plutonium-239 used in the fields of medicine, industry etc.

4. Explain the source of stellar energy. Explain the carbon-nitrogen cycle and proton-proton cycle occurring in stars.

A. Nuclear Fusion:

The process in which two or more lighter nuclei combine to form a heavy nucleus with the emission of energy is called nuclear fusion.

$$4\,{}_1H^1 \rightarrow {}_2He^4 + 2\,{}_{+1}e^1 + 26.7\,\text{MeV}$$

Carbon –Nitrogen Cycle:

Bothe proposed a set of reactions taking place in the central parts of the sun and stars in which carbon and nitrogen act as catalysts.

$${}_6C^{12} + {}_1H^1 \rightarrow {}_7N^{13} + \gamma$$

$${}_7N^{13} \rightarrow {}_6C^{13} + {}_1e^0 + \upsilon$$

$${}_6C^{13} + {}_1H^1 \rightarrow {}_7N^{14} + \gamma$$

$${}_7N^{14} + {}_1H^1 \rightarrow {}_8O^{15} + \gamma$$

$${}_8O^{15} \rightarrow {}_7N^{15} + {}_1e^0 + \upsilon$$

$${}_{7}N^{15} + {}_{1}H^{1} \rightarrow {}_{6}C^{12} + {}_{2}He^{4}$$

The net result of the above processes can be written as

$$4\,{}_{1}H^{1} \rightarrow {}_{2}He^{4} + 2\,{}_{+1}e^{0} + 2\upsilon + \gamma$$

In this process four protons are fused to form a Helium nucleus and two positron releasing energy of about 26.7 MeV and two anti neutrino.

Proton – Proton Cycle:

Bothe and his co-workers suggested the following fusion reactions possible in a star.

$${}_{1}H^{1} + {}_{1}H^{1} \rightarrow {}_{1}H^{2} + {}_{+1}e^{0} + \upsilon ....(a)$$

$${}_{1}H^{1} + {}_{1}H^{2} \rightarrow {}_{2}He^{3} + energy ....(b)$$

The above fusion reactions (a) and (b) must occur twice to yield two ${}_{2}He^{3}$ nuclei. Therefore the next reaction can be written as

$${}_{2}He^{3} + {}_{2}He^{3} \rightarrow {}_{2}He^{4} + 2\,{}_{1}H^{1} + energy$$

The net result of the above reaction is that 4 protons are fused to produce an -particle and a few other particles and release of a total energy 26.7MeV

$$4\,{}_{1}H^{1} \rightarrow {}_{2}He^{4} + 2\,{}_{+1}e^{0} + 2\upsilon + 2\gamma$$

The proton –proton cycle is an important source of energy in the sum and in stars of comparatively lower temperatures (red dwarfs).

PROBLEMS

1. Show that the density of nucleus does not depend upon its mass number (density is independent of mass)?

Sol: Radius of the nucleus $r = r_0\, A^{\frac{1}{3}}$

Volume of the nucleus = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi (r_0 A^{1/3})^3 = \frac{4}{3}\pi r_0^3 A$

Mass of each proton = $1.670020 \times 10^{-27} kg$

Mass of the nucleus = $= 1.670020 \times 10^{-13} A$

Density = $\frac{mass}{volume} = \frac{1.67 \times 10^{-27} \times A}{\frac{4}{3}\pi r_0^3 \times A} \approx 10^{17}$ kg/m3

Hence the density of the nucleus is independent of the mass number A and is the same for all the nuclei.

2. Compare the radii of the nuclei of mass numbers 27 and 64?

Sol: $A_1 = 27, \;\; A_2 = 64$

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{27}{64}\right)^{1/3} = \frac{3}{4}$$

3. The radius of the oxygen nucleus $^{16}_{8}O$ is $2.8 \times 10^{-15} m$. Find the radius of lead nucleus $^{205}_{82}Pb$

Sol: $R_1 = 2.8 \times 10^{-15} m, \; A_1 = 16; \; A_2 = 205, \; R_2 = ?$

$$\frac{R_2}{R_1} = \left(\frac{A_2}{A_1}\right)^{1/3} = \left(\frac{205}{16}\right)^{1/3}$$

$$R_2 = 2.8 \times 10^{-15} \left(\frac{205}{16}\right)^{1/3} = 6.55 \times 10^{-5} m$$

4. Find the binding energy of $^{56}_{26}Fe$. Atomic mass of Fe is 55.9349u and that of Hydrogen is 1.00783u and mass of neutron is 1.00876u

A. Mass of the hydrogen atom m_H = 1.00783u; Mass of neutron m_H = 1.00867 u; Atomic number of iron Z = 26; Mass number of iron A = 56; Mass of iron atom M_a = 55.9349u

Mass defect $\Delta m = [Zm_H+(A\text{-}Z)m_n] - M_a$

$= [\,26 \text{ x } 1.00783+(56\text{-}20)1.00867]\text{-}55.93493 \text{ u} = 0.5287 \text{ u}$

Binding energy = $(\Delta m)c^2 = (0.52878)\,c^2 = (0.52878)(931.5\text{MeV})$

= 492.55 MeV

5. How much energy is required to separate the typical middle mass nucleus $^{120}{}_{50}$ Sn into its constituent nucleons? (Mass of $^{120}{}_{50}$Sn = 119.902199u, mass of proton = 1.007825u and mass of neutron = 1.008665u)

Sol: Mass defect $\Delta m = [Zm_p + (A-Z)m_n - m_N]$

= (50 x 1.007825) + (70 x 1.008665) – 119.902199= 1.095601 amu.

Binding energy = 1.095601 x 931.50 = 1021 MeV.

6. Calculate the binding energy of an α-particle. Given that mass of proton = 1.0073u, mass of neutron = 1.0087u. and mass of α-particle = 4.0015u.

A. m_P = 1.0073u, m_N = 1.0087u, M = 4.0015u

N = A -Z =4 -2 =2 $\left(\because {}_2He^4 = {}_ZX^A\right)$

B.E = Δm x 931.5 MeV

$=\left\{\left[Zm_p+(A-Z)m_n\right]-M\right\}\times 931.5$

$=\left[\left[(2\times 1.0073)+(2\times 1.0087)-4.0015\right]\right]\times 931.5MeV$

= 0.0305 x 931.5 MeV

B.E = 28.4 MeV

7. **Find the energy required to split $^{16}_{8}O$ nucleus into four -particles. The mass of an - particle is 4.002603u and that of oxygen is 15.994915u?**

Sol: $^{16}_{8}O \rightarrow 4 \times _2He^4$

Mass of each α-particle,

$_2He^4 = 4.002603\,amu$

Mass of 4α particles = 4 x 4.00603 = 16.010412 amu

Mass of $^{16}_{8}O = 15.994915\,amu$

Mass defect Δm = 16.010412 – 15.994915 = 0.015497 amu

Energy required = $\Delta m \times 931 MeV$ =0.015497 x 931 = 14.43 MeV.

8. **Calculate the binding energy per nucleon of $^{35}_{17}Cl$ nucleus. Given that mass of $^{35}_{17}Cl$ nucleus = 34.98000u, mass of proton = 1.007825u, mass of neutron = 1.0068665 u and 1 is equivalent to 931MeV?**

Sol: For $_{17}Cl^{35}$, Z = 17, A= 35

$m_p = 1.007825amu$, $m_n = 1.008665amu$, $m_{Cl} = 34.98000amu$

Mass defect

$= \Delta m = (Zm_p + (A-Z)m_n) - M_{Cl}$

[17 (1.007825) + 18(1.008665)] – 34.98000 = 0.30899amu

B.E = 931 x 0.30899 = 287.6743MeV

$$\frac{B.E}{A} = \frac{287.6743}{35} = 8.22MeV$$

9. **Calculate the binding energy per nucleon of $^{40}{}_{20}$Ca. Given that mass of $^{40}{}_{20}$Ca nucleus = 39.962589 u, mass of proton = 1.007825 u. mass of Neutron = 1.008665 u and 1 u is equivalent to 931 MeV?**

A. A =40, Z = 20, A -Z = 20

Δm = $\{Zm_p + (A-Z)m_n\} - M_n$

= { (20 x 1.007825 + (20 x 1.008665)} - 39.962589

= 40.329800 - 39.962589

Δm = 0.367211

Binding energy per nucleon $= \frac{\Delta m \times 931}{A} = \frac{0.367211 \times 931}{40} = 8.547 MeV$.

10. Calculate (i) mass defect, (ii) binding energy and (iii) the binding energy per nucleon of $^{12}{}_{6}C$ nucleus. Nuclear mass of $^{12}{}_{6}C$ = 12.000000 u; mass of proton = 1.007825u and mass of neutron = 1.008665u.

A. A =12, Z =6, N=A -Z =12 - 6= 6

$$\Delta m = \{Zm_p + (A-Z)m_N\} - M_n$$

= { (6 x 1.007825 + (6 x 1.008665)} - 12.000000

= 12.09894 - 12.000000

Δm = 0.09894u

ii) Binding energy = Δm x 931.5 MeV

= 0.09894 x 931.5 = 92.16 MeV

iii) Binding energy per nucleon $= \frac{B.E}{A} = \frac{92.16}{12}$ =7.68MeV

11. The binding energies per nucleon for deuterium and helium are 1.1 MeV and 7.0 MeV respectively. What energy in joules will be liberated when 2 deuterons take part in the reaction?

A. $^{2}_{1}H + ^{2}_{1}H \rightarrow ^{4}_{2}He + Q$

Binding energy per nucleon of helium ($^{4}_{2}He$) =7 MeV

Binding energy = 4 x7 = 28 MeV

Binding energy per nucleon of deuterium ($^{2}_{1}H$) = 1.1 MeV

Binding energy = 2 x 1.1 = 2.2 MeV

Energy liberated (Q) = (28 -(2.2)2] = 23.6 Mev.

i.e. Q = 23.6 x 10^{6} x 1.6 x 10^{-19}

$Q = 37.76 \times 10^{-13} J$

12. Bombardment of lithium with protons gives rise to the following reaction: $^7_3Li + ^1_1H \rightarrow 2\left[^4_2He\right] + Q$. Find the Q-Value of the reaction. The atomic masses of lithium, proton and helium are 7.016u, 1.008u and 4.004u respectively?

A. Mass of lithium = 7.016 amu

Mass of proton = 1.008 amu

Mass of helium = 4.004 amu

Q = [Mass of lithium + mass of proton -(2 x Mass of Helium)] x 931.5

= [7.016 + 1.008 - 2(4.004)] x 931.5 MeV

Q = 0.016 x 931.5 = 14.9 MeV

13. The half life of radium is 1600 years. How much time does 1 g of radium take to reduce to 0.125g?

A. 1g becomes $\frac{1}{2}g$ in one half -life. $\frac{1}{2}$g becomes $\frac{1}{4}$g in another half -life. $\frac{1}{4}$g becomes $\frac{1}{8}g$ = 0.125g in a third half-life.

$\therefore$ Time taken = 3 half -lives = 3 x 1600 = 4800 years.

14. Plutonium decays with a half life of 24,000 years. If plutonium is stored for 72,000 years, what fraction of it remains?

A. T1/2 = 24,000 years

Duration of time (t) = 72,000 years

Number of half lives (n) = $\frac{t}{T_{1/2}}$

$$= \frac{72000}{24000} = 3$$

$\therefore\ 1gm \xrightarrow{1} \frac{1}{2}gm \xrightarrow{2} \frac{1}{4}gm \xrightarrow{3} \frac{1}{8}gm$ $\therefore$ Fraction of plutonium remains = $\frac{1}{8}gm$

15. A certain substance decays to 1/232 of its activity in 25 days. Calculate its half-life?

Sol: $\frac{N}{N_o} = \left(\frac{1}{2}\right)^n$

$$\frac{1}{232} = \left(\frac{1}{2}\right)^n = \frac{1}{2^5}$$

n = 5 days

$$T = \frac{25}{5} = 5days$$

16. The half -life period of a radioactive substance is 20 days. What is the time taken for 7/8th of its original mass to disintegrate?

A. Let the initial mass be one unit.

Mass remaining = $1 - \frac{7}{8} = \frac{1}{8}$

A mass of 1 unit becomes $\frac{1}{2}$ unit in 1 half life

$\frac{1}{2}$ Unit becomes $\frac{1}{4}$ unit in 2nd half life

$\frac{1}{4}$ Unit becomes $\frac{1}{8}$ unit in 3rd half life

$\therefore$ Time taken = 3 half life

= 3 x 20 = 60 days

17. How many disintegrations per second will Occur in one gram of $^{238}_{92}U$, if its half-life against α-decay is 1.42 x 10^{17}s?

A. Given Half -life period (T) = $\frac{0.693}{\lambda}$ = 1.42 10^{17} s

$$\lambda = \frac{0.693}{1.42 \times 10^{17}} = 4.88 \times 10^{-18}$$

Avogadro number (N) = 6.023 x 10^{23} atoms

n = Number of atoms present in 1 gm of $^{238}_{92}U = \frac{N}{A}$

$$= \frac{0.623\times10^{23}}{238} = 25.30 \times 10^{20}$$

Number of disintegrations = $\frac{dN}{dt} = \lambda n = 4.88 \text{ x}10^{-18} \text{ x } 25.30 \text{ x } 10^{20}$

$= 1.2346 \text{ x } 10^{4}$ disintegrates/sec

18. The half-life of a radioactive substance is 100years. Calculate in how many years the activity will decay to 1/10th of its initial value?

A. $T_{1/2} = 100$ years

$$\frac{N}{N_0} = \frac{1}{10} \; ; \; \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{100}$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\Rightarrow \frac{1}{10} = \frac{1}{e^{\lambda t}}$$

$$\Rightarrow e^{\lambda t} = 10 \Rightarrow \lambda t = \log_e^{10}$$

$$t = \frac{2.303 \log_{10}^{10}}{\lambda}$$

$$= \frac{2.303 \times 100}{0.693} = 332.3 \text{ years}$$

19. One gram of radium is reduced by 2milligram in 5 years by *a*-decay**. Calculate the half-life of radium?**

A. Initial mass of radium = 1 gm.

Amount left = 1- 0.002 = 0.998 gm

Time taken t = 5 years

Fraction remained $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n \Rightarrow \frac{0.998}{1} = \frac{1}{2^n} \Rightarrow 2^n = \frac{1000}{998} \Rightarrow n\log 2 = \log 1000 - \log 998$

$$\Rightarrow n = \frac{3-2.9991}{0.3010} = \frac{0.0009}{0.3010} = 0.00299$$

Half life of radius $= T = \frac{t}{n} = \frac{5}{0.00299} = 1672.2 yrs$

20. The half-life of a radioactive substance is 5000 years. In how many years, its activity will decay to 0.2 times of its initial value? Given $\log_{10}5$ = 0.6990?

A. T = 5000 years, $\frac{N}{N_0} = 0.2 = \frac{2}{10} = \frac{1}{5}$

$$\lambda = \frac{0.693}{T} = \frac{0.693}{5000}$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\frac{1}{5} = \frac{1}{e^{\lambda t}} \Rightarrow 5 = e^{\lambda t}$$

$\log_e 5 = \lambda t$

$2.303 \times 0.6990 = \lambda t$

$$t = \frac{2.303 \times 0.6990 \times 5000}{0.693}$$

t = 11614.6years $= 1.1615 \times 10^4$ years

21. An explosion of atomic bomb releases an energy of 7.6 x 10^{13} J. If 200 MeV energy is released on fission of one ^{235}U atom calculate

(i) The number of uranium atoms undergoing fission.

(ii) The mass of uranium used in the atom bomb

A. E = 7.6 x 10^3J

Energy released per fission = 200 MeV

= 200 x 10^6 x 1.6 x 10^{-19}

= 3.2 x 10^{-11} J

Number of uranium atoms (n) = $\frac{\text{Total energy}}{\text{Energy per fission}}$

$$n = \frac{7.6 \times 10^{13}}{3.2 \times 10^{-11}} = 2.375 \times 10^{24} \text{ atoms}$$

Avogadro number (N) = 6.023×10^{23}atoms

Mass of uranium = $\frac{n \times 235}{N} = \frac{2.375 \times 10^{24} \times 235}{6.023 \times 10^{23}}$ = 92.66gms

22. If one microgram of $^{235}_{92}U$ is completely destroyed in an atom bomb, how much energy will be released?

A. m =1 m gm = 10^{-6} gm = 10^{-9} kg

C = 3 x 10^{8} m/s

E = mC^2 = 10^{-9} x (3 x $10^{8})^2$ = 9 x 10^{7} J

23. Calculate the energy released by fission from 2 g of $^{235}_{92}U$ in kWh. Given that the energy released per fission is 200 MeV?

A. Mass of uranium = 2gm

Energy released per fission = 200 MeV

= 200 x 10^{6} x 1.6 x 10^{-19} = 3.2 x 10^{-11} J

Number of atoms in 2 grams of uranium is

$$n = \frac{2 \times 6.023 \times 10^{23}}{235} = 5.125 \times 10^{21} \text{ atoms}$$

Total energy released = No. of atoms x energy released per fission

= 5.125 x 10^{21} x 3.2 x 10^{-11}

= 16.4 x 10^{10}J

$$\text{Energy in Kwh} = \frac{16.4 \times 10^{10}}{36 \times 10^{5}} \text{Kwh}$$

= 0.455 x 10^{5} Kwh

= 4.55 x 10^{4} Kwh

24. 200MeV energy is released when one nucleus of U^{235} undergoes fission. Find the number of fissions per second required for producing a power of 1 megawatt?

A. $1Mw = 10^6 J/s$, Number of atoms = $\frac{10^6}{200\times10^6\times1.6\times10^{-19}}$

$$= \frac{10^{19}}{200\times1.6} = \frac{10\times10^{14}}{2\times1.6} = 3.125\times10^{14}\, atoms$$

25. How much ^{235}U is consumed in a day in an atomic power house operating at 400 MW, provided the whole of mass ^{235}U is converted into energy?

A. Power = 400 MW = 400 x 10^6 W; time = 1 day = 86, 400 s.

Energy produced, E = power time = 400 x 10^6 x 86,400 = 3.456 x 10^{13} J.

As the whole of mass is converted into energy, by Einstein's mass -energy relation

E = Mc^2

$$M = \frac{E}{c^2} = \frac{3.456 \times 10^{13}}{(3 \times 10^8)^2} = 3.84 \times 10^{-4}\ kg = 0.384\ g.$$