

CURRENT ELECTRICITY

Important Points:

1. Electric Current:

The charge flowing any cross-section per unit time in a conductor is called electric current.

$$\text{Electric Current } I = \frac{q}{t}$$

2. Current Density:

a) The current flowing normally through unit area of cross-section is called current density.

b) Current density is a vector and is along the motion of positive charge at that point.

$$\text{Current density (J)} = \frac{i}{A} = nev_d \text{ amp / m}^2$$

3. Ohm'S Law:

At constant temperature, the potential difference across the conductor is proportional to the current passing through the conductor.

$$V = IR \text{ (or) } R = \frac{V}{I} \text{ Where R is called the Electrical Resistance.}$$

4. Resistance:

The ratio of potential difference across the ends of a conductor and the current passing through the conductor is said to be the resistance of the conductor.

5. Resistance in Series:

$$R = R_1 + R_2 + R_3$$

The effective resistance is equal to the sum of individual resistances.

6. Resistance in Parallel:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

The reciprocal of effective resistance is equal to the sum of reciprocals of individual resistances.

7. Temperature Coefficient of Resistance (α):

a) If R_0 and R_t are the resistances of the conductor at $0^\circ C$ and $t^\circ C$ respectively, then the

temperature coefficient of resistance is given by $\alpha = \frac{R_t - R_0}{R_0 t} / ^\circ C$

b) If R_1 and R_2 are the resistances of the conductor at $t_1^\circ C$ and $t_2^\circ C$ then $\alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1} / ^\circ C$

8. Electromotive Force:

The work done in moving a unit positive charge completely round the closed circuit including the battery is called emf.

9. Terminal voltage and loss voltage $V = E - ir$

10. Cells in Series:

Current through the circuit = $I = \frac{nE}{R + nr}$

11. Cells in Parallel:

Current through the circuit (I) = $\frac{mE}{(r + mR)}$

12. Potential Difference:

a) The work done in moving a unit positive charge between any two points in a circuit is called potential difference

b) Potential difference is a scalar quantity.

13. Terminal voltage and loss voltage $V = E - ir$

14. Cells in Series:

$$\text{Current through the circuit} = I = \frac{nE}{R + nr}$$

15. Cells in Parallel:

$$\text{Current through the circuit (I)} = \frac{mE}{(r + mR)}$$

16. Kirchoff'S Laws:

First Law: It states that the algebraic sum of the currents meeting at any Junction in an electric circuit is zero.

Second Law:

It states that in any closed mesh of a circuit, the algebraic sum of the products of the current and resistance in each part of the loop is equal to the algebraic sum of the emf's in that loop.

17. Wheatstone Bridge:

Wheat Stones Bridge is used to compare the resistances, to determine unknown resistance and to measure small strain in hard materials. This works on the principle of Kirchhoff's laws.

$$\frac{P}{Q} = \frac{R}{S} \text{ This is the whetstone's bridge balancing condition.}$$

18. Meter Bridge: $X = \left(\frac{l_1}{100 - l_1} \right) R$

19. Comparison Of Emfs Using Potentiometer:

a) l_1 and l_2 are balancing lengths when two cells of emfs, E_1 and E_2 are connected

in the secondary circuit $\frac{E_1}{E_2} = \frac{l_1}{l_2}$

b) By sum and difference method, $\frac{E_1 + E_2}{E_1 - E_2} = \frac{l_1}{l_2}$, $\frac{E_1}{E_2} = \frac{l_1 + l_2}{l_1 - l_2}$

c) Internal resistance of a cell $r = \left(\frac{l_1 - l_2}{l_2} \right) R$

Very Short Answer Questions

1. Define Mean Free Path of electron in a conductor?

A. Mean Free Path:

The average distance traveled by an electron between two successive collisions is called Mean Free Path.

2. State Ohm's law and write its mathematical form?

A. Ohm's Law:

At constant temperature, the current passing through a conductor is directly proportional to the potential difference between its ends.

Mathematical Form:

If 'V' is the potential difference between the ends of a conductor and 'I' is the current through it. Then

$$V = iR$$

Where 'R' is electric resistance of the conductor

3. Define resistivity or specific resistance?

A. Resistivity (ρ):

The resistance 'R' of conductor is directly proportional to its length 'l' and inversely proportional to its area of cross section 'A'

$$\text{Resistance } R \propto \frac{l}{A} \text{ or } R = \rho \frac{l}{A}$$

Where ρ is called specific resistance or resistivity of the material.

Definition:

Resistivity is numerically equal to the resistance of a conductor of unit length and unit area of cross section.

4. Define temperature coefficient of resistance?

A. Temperature coefficient of Resistance:

It is defined as the ratio of change in the resistance of a conductor to its original resistance for 1°C rise in its temperature.

$$\text{Temperature coefficient of resistance } \alpha = \frac{R_1 - R_0}{R_0 t} \text{ } ^\circ\text{C}$$

5. Under what conditions is the current through the mixed grouping of cells maximum?

A. Current flowing in the circuit is
$$I = \frac{nE}{R + \frac{nr}{m}} = \frac{mnE}{mR + nr}$$

In order to obtain the maximum current, the cells should be mixed grouped such that external resistance in the circuit is equal to the total internal resistance of the cells.

$$\text{i.e. } mR = nr \quad R = \frac{nr}{m}$$

$$\text{Then } I_{\max} = \frac{mE}{2r}$$

$$I_{\max} = \frac{nE}{2R}$$

6. If a wire is stretched to double its original length without loss of mass, how will the resistivity of the wire be influenced?

A. The resistivity of the wire depends on the nature of material and independent of dimensions of the conductor. The increase in length of the wire will not affect its resistivity.

7. Why is manganin, used for making standard resistors?

A. For alloys such as manganin, constantan and nichrome temperature coefficient of resistance is negligibly small and resistivity is high, hence these are used to make resistance wires or standard resistances.

8. The sequence of bands marked on a carbon resistor are: Red, Red, Red, Silver. What is its resistance and tolerance?

A. Resistance: $22 \times 10^2 \Omega$

Tolerance: 10%

9. Write the color code of a carbon resistor of resistance 23 kilo Ohms?

A. $R=23$ Kilo Ohms = 23×10^3 Ohms

Color code: Red, Orange, Orange

10. If the voltage 'V' applied across a conductor is increased to 2V, how will the drift velocity of the electrons change?

A. Drift velocity $v_d = \frac{eE}{m} \tau$

If 'l' is the length of the conductor and 'V' is the potential difference across it, then

$$v_d = \frac{e}{m} \left[\frac{V}{l} \right] \tau$$

$$v_d = \frac{eV}{ml} \tau$$

$\therefore v_d \propto V$, i.e. if potential difference increased from V to 2V, the drift velocity of the electrons will be doubled.

11. Two wires of equal length, of copper and manganin, has the same resistance. Which wire is thicker?

A. Let ρ_1 and ρ_2 be resistivities of copper and manganin and A_1 and A_2 be the areas of cross-section of the wires of copper and manganin respectively. Given two wires of equal lengths and have the same resistance

$$R_1 = R_2$$

$$\frac{\rho_1 l}{A_1} = \frac{\rho_2 l}{A_2} \Rightarrow \frac{A_1}{A_2} = \frac{\rho_1}{\rho_2}$$

$$\left[\begin{array}{l} \rho_{copper} = 1.7 \times 10^{-8} \Omega m \\ \rho_{manganin} = 44 \times 10^{-8} \Omega m \end{array} \right]$$

Since the resistivity of copper (ρ_1) is less than that of manganin (ρ_2), so area of cross-section of copper is less than that of manganin wire. Thus manganin wire is thicker.

12. Why are household appliances connected in parallel?

A. Most of the household appliances need constant voltage to run and consume current according to the power required. Only in parallel connection the voltage is constant. Hence household appliances are connected in parallel.

13. The electron drift speed in metals is small ($\sim ms^{-1}$) and the charge of the electron is also very small ($\sim 10^{-19}C$), but we can still obtain a large amount of current in a metal. Why?
- A. It is because the number of electrons per unit volume is very large ($\approx 10^{29} / m^3$).

Short Answer Questions

1. A battery of emf 10V and internal resistance 3Ω is connected to a resistor R.
- (i) If the current in the circuit is 0.5 A. Calculate the value of R.
- (ii) What is the terminal voltage of the battery when the circuit is closed?

A. $E=10V$ and $r = 3\Omega$

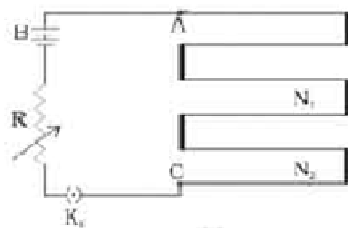
$$i) i = \frac{E}{(R+r)} \Rightarrow 0.5 = \frac{10}{(R+3)}$$

$$\therefore R = 17\Omega$$

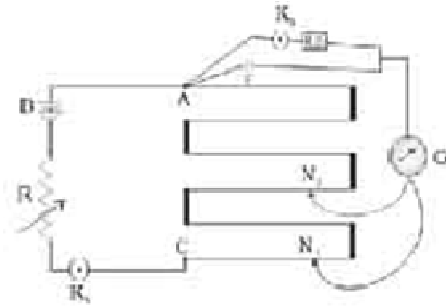
$$ii) \text{Terminal voltage } V = E - ir = 10 - (0.5) \times 3 = 8.5V$$

2. Draw a circuit diagram showing how a potentiometer may be used to find internal resistance of a cell and establish a formula for it?
- A. When current through potentiometer wire and resistance per unit length of potentiometer wire are constant, then potential difference across a length of potentiometer wire is directly proportional to its length.

Principle of potentiometer is, when i and ρ are constant, then $V \propto l$



Determination of internal resistance of a cell by using potentiometer:



We can use a potentiometer to measure internal resistance of a cell. For this the cell (emf ϵ) whose internal resistance (r) is to be determined is connected across a resistance box through a key K_2 , as shown in the figure. With key K_2 open, balance is obtained at length $l_1 (AN_1)$. Then,

$$\epsilon = \phi l_1 \text{----- (1)}$$

When key K_2 is closed, the cell sends a current (I) through the resistance box (R). If V is the terminal potential difference of the cell and balance is obtained at length $l_2 (AN_2)$,

$$V = \phi l_2 \text{----- (2)}$$

So, we have $\frac{\epsilon}{V} = \frac{l_1}{l_2} \text{----- (3)}$

But, $\epsilon = I(r + R)$ and $V = IR$. This gives

$$\frac{\epsilon}{V} = \frac{(r + R)}{R} \text{----- (4)}$$

From equations (3) and (4) we have

$$\frac{(R + r)}{R} = \frac{l_1}{l_2} = r = R \left(\frac{l_1}{l_2} - 1 \right) \text{----- (5)}$$

Using equation (5) we can find the internal resistance of a given cell.

The potentiometer has the advantage that it draws *no current* from the voltage source being measured. As such it is unaffected by the internal resistance of the source.

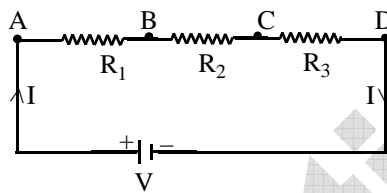
3. Derive an expression for the effective resistance when three resistors are connected in
(i) Series Combination (ii) Parallel Combination

A. (i) Series Combination:

Consider three resistors R_1, R_2 and R_3 connected in series to a cell of emf V . Since the three resistances are in series, same current flows all the resistances. Let V_1, V_2 and V_3 be the potential difference across the three resistors respectively.

$$V_1 = IR_1, V_2 = IR_2 \text{ and } V_3 = IR_3.$$

$$\text{But } V = V_1 + V_2 + V_3, \quad V = IR_1 + IR_2 + IR_3$$



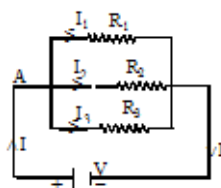
If equivalent resistance of this series combination is R , then

$$V = IR = I(R_1 + R_2 + R_3) \quad \text{or} \quad R = R_1 + R_2 + R_3$$

Thus, equivalent resistance of a series combination of resistors is equal to sum of resistances of all resistors.

(ii) Parallel Combination:

Consider three resistors R_1, R_2 and connected in parallel to a cell of emf V . Since the three resistors are parallel, the potential difference across cell resistor is same series V . Let i_1, i_2 and i_3 be the current through the resistors respectively.



$$I_1 = V/R_1, I_2 = V/R_2 \text{ and } I_3 = V/R_3.$$

$$\text{But, } I = I_1 + I_2 + I_3 \quad \text{or} \quad I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \quad \text{Or} \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Thus the reciprocal of effective resistance is equal to the sum of reciprocals of individual resistances.

4. 'm' cells each of emf E and internal resistance 'r' are connected in parallel .What is the total emf and internal resistance ?Under what conditions is the current drawn from mixed grouping of cells a maximum?

A. **Parallel Combination:**

In parallel combination of identical cells, the effective emf in the circuit is equal to the emf due to a single cell i.e. E. The total internal resistance is r / m .

Mixed Grouping:

The current in the external resistance R is given in a mixed grouping of cells is given by

$$I = \frac{mnE}{mR + nr}$$

The current I will be maximum if $(mR + nr)$ is minimum, i.e.

$$\text{Or } mR = nr \text{ or } R = \frac{nr}{m}.$$

i.e. external resistance = total internal resistance of all the cells.

5. Define electric resistance and write its S.I unit. How does the resistance of a conductor vary if
- (a) Conductor is stretched to 4 times of its length.
 - (b) Temperature of conductor is increased.

A. **Resistance:**

Resistance of a conductor is defined as the ratio of the potential difference 'V' across the conductor to the current 'i' flowing through the conductor.

$$\text{Resistance (R)} = \frac{V}{i}$$

S.I unit of Resistance: volt/ampere (or) Ohm

a) The resistance of a conductor is given by

$$R = \rho \frac{l}{A}$$

If a conductor is stretched or elongated, then the volume of the conductor is constant

Given that conductor is stretched to 4 times of its length

$$R = \rho \frac{l}{V/l} \quad (V = A \times l)$$

$$R = \rho \frac{l^2}{V} \Rightarrow R \propto l^2$$

Let, $l_1 = l$, $l_2 = 4l$

$$R_1 = R, \quad R_2 = ?$$

$$\frac{R_2}{R_1} = \left(\frac{l_2}{l_1}\right)^2 = \left(\frac{4l}{l}\right)^2 = \frac{16l^2}{l^2}$$

$$R_2 = 16R_1$$

Resistance of a conductor increases by 16 times that of original resistance.

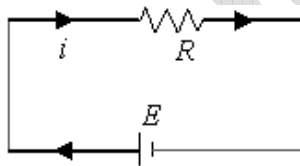
b) If the temperature is increased the resistance of the conductor also increases.

6. When the resistance connected in series with a cell is halved, the current is equal to or slightly less or slightly greater than double. Why?

A. Current $(i) = \frac{E}{r + R}$

Given, $R = \frac{R}{2}$, then $i^1 = \frac{E}{r + R^1}$

$$i^1 = \frac{E}{r + R/2}$$



$$i^1 = \frac{2E}{2r + R}$$

So the current is slightly less than double.

If $r = 0$, $i^{11} = \frac{E}{R/2}$

$$i^{11} = \frac{2E}{R}$$

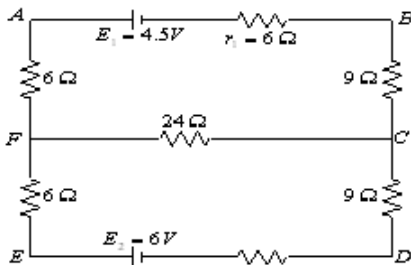
So the current is slightly greater than double.

7. Two cells of emfs 4.5V and 6.0V and internal resistance 6Ω and 3Ω respectively have their negative terminals joined by a wire of 18Ω and positive terminals by a wire of 12Ω resistance. A third resistance wire of 24Ω connects middle points of these wires. Using Kirchhoff's laws, find the potential difference at the ends of this third wire?

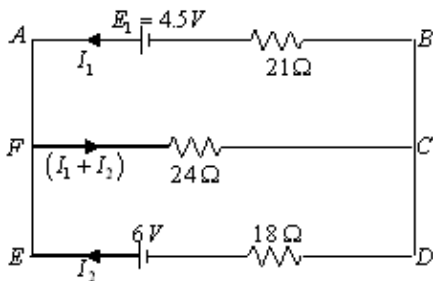
A. Given positive terminals of the cells 4.5 V and 6 V are connected to 12Ω resistance and negative terminals of cell connected to 18Ω . The resistance of is connected between middle point of these wires.

$$R_1 = R_2 = \frac{12}{2} = 6\Omega$$

$$R_3 = R_4 = \frac{18}{2} = 9\Omega$$



The given circuit is equivalent to the circuit shown below



Applying Kirchhoff's second law to the closed loop ABCFA

$$21I + 24(I_1 + I_2) = 4.5$$

$$45I_1 + 24I_2 = 4.5 \text{ ----- (1)}$$

Applying Kirchhoff's second law to closed loop CDEFC

$$18I_2 + 24(I_1 + I_2) = 6$$

$$24I_1 + 42I_2 = 6 \text{ ----- (2)}$$

Solving equation (1) and (2)

$$I_1 = \frac{7.5}{219} = 0.0342A$$

$$I_2 = \frac{189}{1533} = \frac{27}{219} = 0.1233A$$

Therefore the current through the resistance R,

$$I_1 + I_2 = \frac{7.5}{219} + \frac{27}{219} = \frac{34.5}{219}$$

∴ Potential difference across the resistor 'R'

$$= (I_1 + I_2)R$$

$$= \frac{34.5}{219} \times 24 = \frac{828}{219} = 3.78V$$

8. Three resistors each of resistance 10 ohm are connected, in turn, to obtain
(i) Minimum Resistance (ii) Maximum Resistance. Compute

(a) The effective resistance in each case

(b) The ratio of minimum to maximum resistance so obtained.

A. (a) The effective resistance is minimum, when resistors are connected in parallel.

$$\therefore R_p = \frac{R}{n} = \frac{10}{3} = 3.33\Omega$$

The effective resistance is maximum, when resistors are connected in series.

$$\therefore R_s = nR = 3 \times 10 = 30\Omega$$

$$(b) \frac{R_{\min}}{R_{\max}} = \frac{1}{n^2} = \frac{1}{9}$$

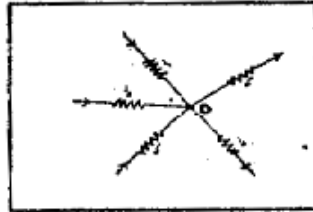
9. State Kirchoff's law for an electrical net work. Using these laws deduce the condition for balance in a Wheatstone bridge?

A. First Law:

The sum of the currents flowing towards a Junction is equal to the sum of the currents flowing away from the Junction.

Explanation:

Let five conductors carrying currents are meeting at a Junction 'O' of an electric. Let i_1, i_2 and i_3 be the currents flowing towards the Junction and the currents i_4 and i_5 flowing from the Junction. Then



Kirchhoff's first law

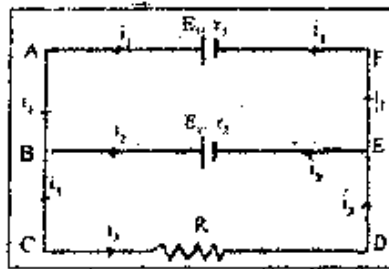
$$i_1 + i_2 + i_3 = i_4 + i_5$$

Second Law:

In any closed mesh of a circuit, the algebraic sum of the products of the current and resistance in each part of the loop is equal to the algebraic sum of the emf's in that loop.

Explanation:

Consider the circuit as shown in the figure.



In the closed loop ACDF A, $E_1 = i_1 r_1 + i_3 R$

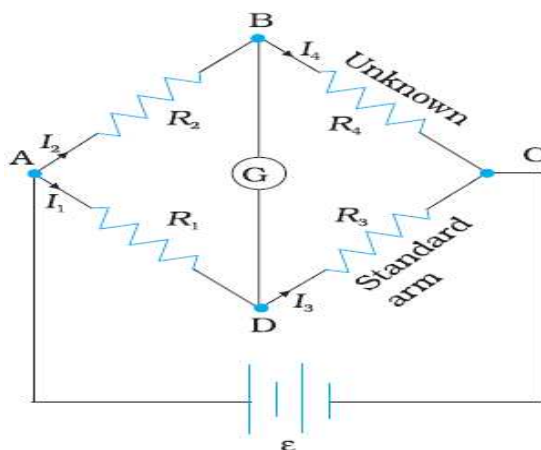
In the closed loop BCDE B, $E_2 = i_2 r_2 + i_3 R$

In the closed loop ABEFA, $E_1 - E_2 = i_1 r_1 - i_2 r_2$

Wheatstone Bridge:

Wheatstone bridge is used to compare the resistances, to determine unknown resistance and to measure small strain in hard materials. This works on the principle of Kirchhoff's laws.

Description:



Wheatstone bridge consists of four resistors R_1, R_2, R_3 and R_4 connected in the four arms of a square to form four Junctions A, B, C and D as shown in the figure. A galvanometer G is connected between the Junction B and D. A battery of emf E and no internal resistance is connected across the Junction A and C. Let G be the resistance of the galvanometer.

Principle:

The currents in the resistances are shown and let i_g be the current passing through the galvanometer. Consider the case when the current through the galvanometer is zero i.e. $i_g = 0$. This is called bridge balancing condition.

By applying Kirchhoff's first law to the Junction B and D

$$i_1 = i_3 \quad \text{and} \quad i_2 = i_4$$

By applying Kirchhoff's 2nd law to the closed loop ADBA,

$$-I_1 R_1 + 0 + I_2 R_2 = 0 \Rightarrow \frac{I_1}{I_2} = \frac{R_2}{R_1} \text{ ---- (1)}$$

By applying Kirchhoff's 2nd law to the closed loop CBDC,

$$I_2 R_4 + 0 - I_1 R_3 = 0 \Rightarrow \frac{I_1}{I_2} = \frac{R_4}{R_3} \text{ ---- (2)}$$

From equations (1) and (2), $\frac{R_2}{R_1} = \frac{R_4}{R_3}$

This is called the bridge balancing condition.

10. State the working principle of potentiometer. Explain with the help of a circuit diagram how the emf of two primary cells is compared by using the potentiometer?

A. Description:

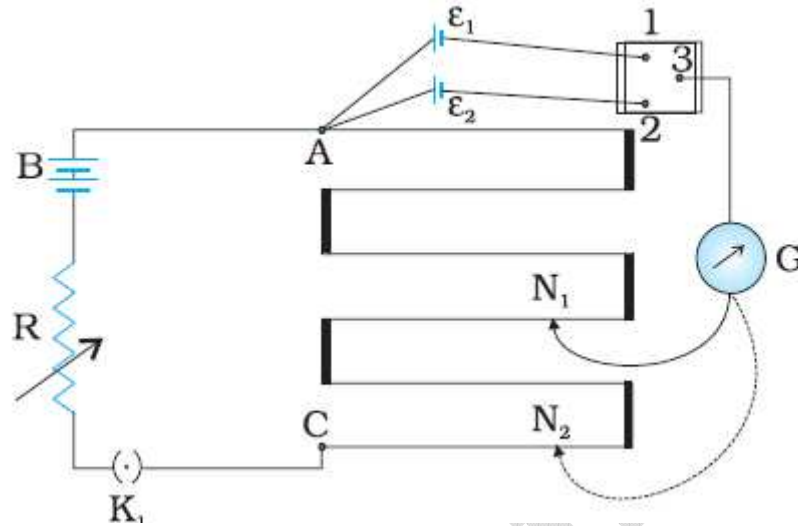
A potentiometer consists of uniform wire of length 10m arranged between A and C as 10 wires each of length 1m on a wooden board. The ends are connected by thick metal strips. The wire has specific resistance and low temperature coefficient of resistance. The resistance of the total wire of the potentiometer is about 5Ω . The balancing length is measured from the end which is connected to the positive terminal of the battery by moving the Jockey J on the wire.

Principle: An unknown emf (or) potential difference is compared with known variable potential difference produced on the potentiometer wire. A current I flows through the wire

which can be changed by the rheostat. Since the wire is uniform the p.d between A and any point at a distance l from A is

$$\varepsilon(l) = \phi l \text{ Where } \phi \text{ is the potential drop per unit length of the potentiometer wire.}$$

Comparison of emf two cells:



The circuit used to compare the emf of two cells is shown in the figure. The points marked 1,2 ,3 form a two way plug key. Initially the 1 and 3 of the key are connected, so that the galvanometer is connected to ε_1 . The Jockey is moved along the wire till the balance point is obtained at N_1 at a distance l_1 from A.

By applying Kirchoff's 2nd law to the closed loop AN_1G31A ,

$$\phi l_1 + 0 - \varepsilon_1 = 0 \quad \text{---- (1)}$$

Similarly for the another cell of emf ε_2 is balanced against l_2 is measured. Hence

$$\phi l_2 + 0 - \varepsilon_2 = 0 \quad \text{---- (2)}$$

From equations (1) and (2), $\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$

Thus emf of two primary cells can be compared.

Precautions:

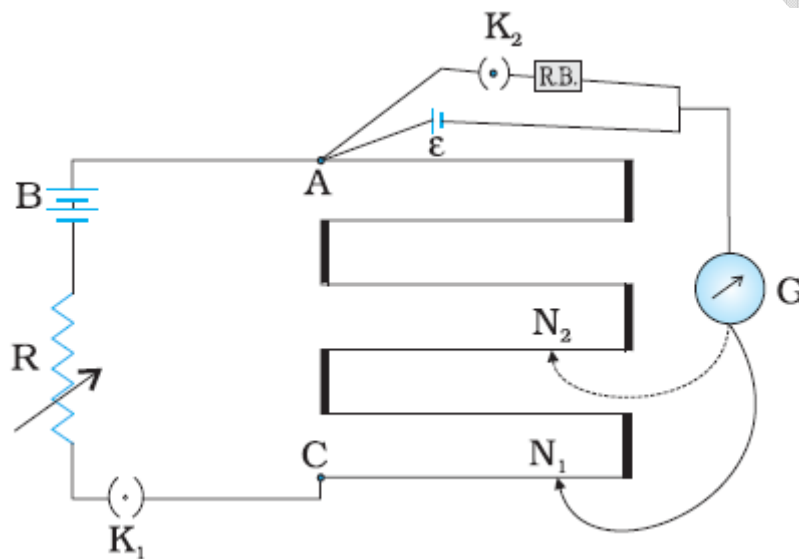
1. Jockey should not be dragged along the wire.
2. The current value should be as small as possible.
3. Current should be passed only while taking the readings.

11. State the working principle of potentiometer explain with the help of a circuit diagram how the potentiometer is used to determine the internal resistance of the given primary cell ?

A. Description:

A potentiometer consists of uniform wire of length 10m arranged between A and C as 10 wires each of length 1m on a wooden board. The ends are connected by thick metal strips. The wire has specific resistance and low $\epsilon(l) = \phi l$ where ϕ is the potential drop per unit length of the potentiometer wire.

To find the internal resistance of a cell:



The circuit used to find the internal resistance of a cell is shown. A cell of emf ϵ whose internal resistance (r) is to be determined is connected across a resistance box through a plug key k_2 . With the key k_2 is open, the balancing length l_1 is measured. $\epsilon = \phi l_1 \dots (1)$

Now the key k_2 is closed so that RB is included in the secondary circuit and the balancing point l_2 is the measured.

$$V = \phi l_2 \dots (2)$$

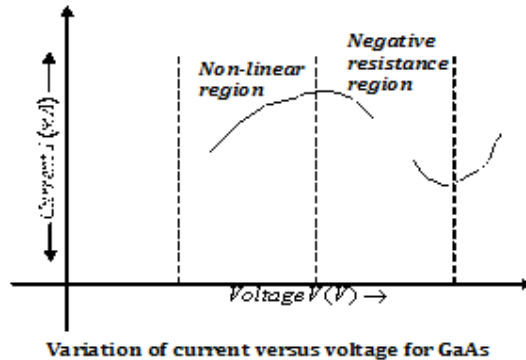
From (1) and (2) $\frac{\epsilon_1}{V} = \frac{l_1}{l_2}$ But $\epsilon = I(r + R)$ and $V = IR \therefore \frac{(R + r)}{R} = \frac{l_1}{l_2} \Rightarrow r = \left(\frac{l_1 - l_2}{l_2} \right) R$

Precautions:

1. Jockey should not be dragged along the wire.
2. The current value should be as small as possible.
3. Current should be passed only while taking the readings.

12. Show the variation of current versus voltage graph for GaAs and mark the (i) Non-Linear Region (ii) Negative Resistance Region

A. The relation between V and I is not unique, i.e., there is more than one value of V for the same current I (fig.). A material exhibiting such behavior is GaAs.



13. A student has two wires of iron and copper of equal length and diameter. He first joins two wires in series and passes an electric current through the combination which increases gradually. After that he joins two wires in parallel and repeats the process of passing current. Which wire will glow first in each case?

A. Given two wires are iron and copper of equal length and diameter.

$$\therefore \text{The resistivity of iron } (\rho_{Fe}) = 10 \times 10^{-8} \Omega m$$

$$\text{The resistivity of copper } (\rho_{Cu}) = 1.7 \times 10^{-8} \Omega m$$

We know $R = \frac{\rho l}{A}$

$$R \propto \rho \quad [\because \text{Given are constants}]$$

a) If two wires are connected in series, the current through both of them remain same. So effective resistance increases

Since

$$Q \propto R \quad \therefore \text{So more resistance wire will glow first i.e. Iron}$$

b) If two wires are connected in parallel, the voltage across both of them is same. So effective resistance decreases.

$$\text{Since } Q = \frac{V^2}{R} t$$

$$Q \propto \frac{1}{R} \quad [\text{Since potential difference is constant}]$$

So less resistance wire will glow first i.e. copper.

14. Three identical resistors are connected in parallel and total resistance of the circuit is $R/3$. Find the value of each resistance?

A. Given three resistors are identical.

$$R_1 = R', R_2 = R', R_3 = R'$$

If they are connected in parallel

$$\frac{1}{R_p} = \frac{1}{R'} + \frac{1}{R'} + \frac{1}{R'} \Rightarrow \frac{1}{R_p} = \frac{3}{R'} \quad \therefore R_p = \frac{R'}{3}$$

$$\text{Given, } R_p = \frac{R}{3} \quad \frac{R}{3} = \frac{R'}{3} \Rightarrow R' = R$$

Long Answer Questions

1. Under what condition is the heat produced in an electric circuit (a) Directly Proportional to (b) Inversely Proportional to the resistance of the circuit? Compute the ratio of the total quantity of heat produced in the two cases.

A. According to Joule's law $W = JH$ and $H = \frac{i^2 R t}{J} = \frac{V^2 t}{JR}$

(a) If the resistances in a circuit are in series, same current flows through them. Hence when 'i' is constant, $H_1 \propto R_s$

(b) If the resistances in a circuit are in parallel, p.d. across all the resistances is same. Hence when V is constant, $H_2 \propto \frac{1}{R_p}$

1st case, $H_1 = \frac{i^2 R t}{J} = \left(\frac{E}{R_s}\right)^2 \times \frac{R_s t}{J}$

$$H_1 \propto \frac{1}{R_s}$$

2nd case, $H_2 = \frac{V^2 t}{RJ} = \frac{E^2 t}{R_p t}$

$$\therefore \frac{H_1}{H_2} = \frac{R_p}{R_s}$$

Where R_s and R_p are the effective resistances in series and parallel combination

2. Two metallic wires A and B are connected in parallel. Wire A has length L and radius r wire B has a length 2L and radius 2r. Compute the ratio of the total resistance of the parallel combination and resistance of wire A?

A: Given, for wire A

$$\text{Length } (l_1) = L, \text{ radius } (r_1) = r$$

For wire B

$$\text{length } (l_2) = 2L, \text{ radius } (r_2) = 2r$$

$$\therefore \text{The resistance of wire A } (R_A) = \frac{\rho l_1}{\pi r_1^2}$$

$$= \frac{\rho l}{\pi r^2}$$

The resistance of wire B (R_B) = $\frac{\rho l_2}{\pi r_2^2}$

$$= \frac{\rho(2l)}{\pi r_2^2}$$

$$= \frac{\rho(\cancel{2}l)}{\pi(\cancel{4}r^2)} = \frac{1}{2} \frac{\rho L}{\pi r^2}$$

$$R_B = \frac{1}{2} R_A$$

Given two wires are connected in parallel. So, total resistance of the combination is

$$\frac{1}{R_p} = \frac{1}{R_A} + \frac{1}{R_B} = \frac{1}{R_A} + \frac{2}{R_A}$$

$$\frac{1}{R_p} = \frac{3}{R_A}$$

$$\therefore R_p = \frac{R_A}{3}$$

\therefore The ratio of total resistance of the combination and resistance of wire A is

$$\frac{R_p}{R_A} = \frac{R_A/3}{R_A} = \frac{1}{3}$$

$$\therefore R_p : R_A = 1 : 3$$

3. In a house three bulbs of 100W each are lighted for 4 hours daily and six tube lights of 20 W each are lighted for 5 hours daily and a refrigerator of 400 W is worked for 10 hours daily for a month of 30 days. Calculate the electricity bill if the cost of one unit is Rs.4.00?

A . Electric energy consumed by 3 bulbs at the rate of 4 hours per day for 30 days

$$= \text{Total Wattage} \times \text{hours of use} \times 30$$

$$= (3 \times 100) \times 4 \times 30 \text{ watt hours} = \frac{36000}{1000} \text{ KWH} = 36 \text{ KWH} = 36 \text{ Units}$$

Electric energy consumed by 6 tube lights at the rate of 5 hours per day for 30 days

$$= \text{Total Wattage} \times \text{hours of use} \times 30$$

$$= (6 \times 20) \times 5 \times 30 \text{ watt hours} = \frac{18000}{1000} \text{ KWH} = 18 \text{ KWH} = 18 \text{ Units}$$

Electric energy consumed by refrigerator at the rate of 10 hours per day for 30 days

$$= \text{Total Wattage} \times \text{hours of use} \times 30$$

$$= (400) \times 10 \times 30 \text{ watt hours} = \frac{120000}{1000} \text{ KWH} = 120 \text{ KWH} = 120 \text{ Units}$$

$$\text{Total unit} = 36 + 18 + 120 = 174$$

$$\text{Cost of 1 unit} = 4 \text{ rupees}$$

$$\text{Cost of 174 units} = 174 \times 4 = \text{Rs } 696/-$$

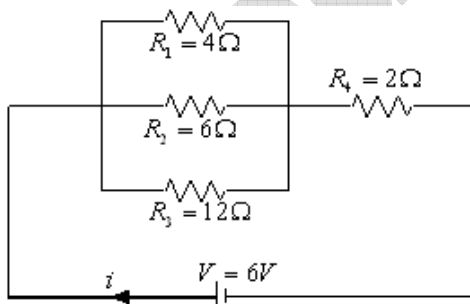
4. Three resistors of 4 ohms, 6 ohms and 12 ohms are connected in parallel. The combination of above resistors is connected in series to a resistance of 2 ohms and then to a battery of 6 volts. Draw a circuit diagram and calculate?

a) Current in main circuit

b) Current flowing through each of the resistors in parallel

c) Potential difference and the power used by the 2 ohm resistor.

A. $R_1 = 4\Omega, R_2 = 6\Omega, R_3 = 12\Omega, R_4 = 2\Omega, V = 6V$



The effective resistance of parallel combination is

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_p} = \frac{1}{4} + \frac{1}{6} + \frac{1}{12}$$

$$= \frac{6+4+2}{24} = \frac{12}{24}$$

$$R_p = \frac{24}{12} = 2\Omega$$

Here, R_p and R_4 are in series. So the effective resistance of the combination

$$R_p + R_4 = 2 + 2 = 4\Omega$$

a) The current in the main circuit is

$$i = \frac{V}{R_p + R_4} = \frac{6}{4} = 1.5A$$

b) Current flowing through each of the resistors in parallel is

$$i = \frac{1.5A}{3} = 0.5A$$

c) Potential difference across the 2 ohm resistor

$$V = iR$$

$$= 1.5 \times 2$$

$$V = 3V$$

$$\text{Power (P)} = V \times i$$

$$= 3 \times 1.5$$

$$\text{Power} = 4.5W$$

5. Two lamps, one rated 100 w at 220 V and the other 60 w at 220 V are connected in parallel to a 220 V supply. What current is drawn from the supply line?

A. Resistance of the first lamp $R_1 = \frac{V^2}{P} = \frac{220 \times 220}{100}$ ohms.

Resistance of the second lamp $R_2 = \frac{V^2}{P} = \frac{220 \times 220}{60}$ ohms

Effective resistance $R_{eff} = \frac{R_1 R_2}{R_1 + R_2} = \frac{220 \times 220}{160}$ ohms

Current drawn from the supply line $I = \frac{V}{R} = \frac{220 \times 160}{220 \times 220} = 0.73 A$

6. Estimate the average drift speed of conduction electrons in a copper wire of cross sectional area $3.0 \times 10^{-7} \text{ m}^2$ carrying a current of 5A. Assume that each copper atom contribute roughly one conduction electron. The density of copper is $9.0 \times 10^3 \text{ kg / m}^3$ and its atomic mass is 63.5u?

A. $A = 3.0 \times 10^{-7} \text{ m}^2$; $i = 5 \text{ A}$; $\rho = 9.0 \times 10^3 \text{ kg / m}^3$; $m = 63.5 \text{ u}$

$$n = \frac{9.0 \times 10^3}{63.5 \times 1.6 \times 10^{-27}} = 8.85 \times 10^{28}$$

$$v_d = \frac{i}{neA} = \frac{5}{8.85 \times 10^{28} \times 1.6 \times 10^{-19} \times 3 \times 10^{-7}}$$

$$\therefore v_d = 0.12 \text{ mm / s}$$

7. Compare the drift speed obtained above with

i) Thermal speed of copper atoms at ordinary temperatures.

ii) Speed of propagation of electric field along the conductor which causes the drift motion.

A. (b) (i) At a temperature T, the thermal speed of a copper atom of mass M is obtained from $[\langle (1/2) M v^2 \rangle = (3/2) k_B T]$ and is thus typically of the order of $\sqrt{k_B T / M}$, where k_B is the Boltzmann constant. For copper at 300 K, this is about $2 \times 10^7 \text{ m/s}$. This figure indicates the random vibrational speeds of copper atoms in a conductor. Note that the drift speed of electrons is much smaller, about times the typical thermal speed at ordinary temperatures.

(ii) An electric field travelling along the conductor has a speed of an electromagnetic wave, namely equal to $3.0 \times 10^8 \text{ m s}^{-1}$. The drift speed is, in comparison, extremely small; smaller by a factor of 10^{-11}

PROBLEMS

1. A 10 thick wire is stretched so that its length becomes three times. Assuming that there is no change in its density on stretching? Calculate the resistance of the stretched wire?

Sol: $R_1 = 10\Omega$ $l_2 = 3l_1$

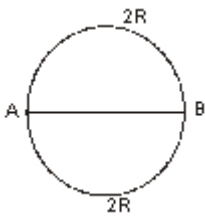
$$R = \frac{sl}{A} = \frac{sl \times l}{A \times l} = \frac{sl^2}{v}$$

$$R \propto l^2 \Rightarrow \frac{R_2}{R_1} = \frac{l_2^2}{l_1^2} = \frac{9l_1^2}{l_1^2} = 9$$

$$R_2 = 9R_1 = 9 \times 10 = 90\Omega$$

2. A wire of resistance $4R$ is bent in the form of circle. What is the effective resistance between the ends of the diameter?

Sol:



$$R_1 = R_2 = 2R$$

$$R_{AB} = \frac{R_1 R_2}{R_1 + R_2} = \frac{2R(2R)}{4R} = R$$

3. Find the resistivity of a conductor which carries a current of density of $2.5 \times 10^6 \text{ A m}^{-2}$ when an electric field of 15 Vm^{-1} is applied across it?

Sol: $j = 2.5 \times 10^6 \text{ Am}^{-2}$

$$E = 15 \text{ Vm}^{-1}$$

$$\rho = \frac{E}{J} = \frac{15}{2.5 \times 10^6} = \frac{150}{25} \times 10^{-6}$$

$$= 6 \times 10^{-6} \Omega \text{ m}$$

4. What is the color code for a resistor of resistance $350\text{m}\Omega$ with 5% tolerance?

Sol: $R = 350\text{ m}\Omega = 350 \times 10^{-3} \Omega$

$$R = 35 \times 10^{-2} \Omega$$

3 → Orange.

5 → Green.

10^{-2} → Silver

5% → Gold.

5. You are given 8Ω resistor? What length of wire of resistivity should be joined in parallel with it to get a value of 6Ω ?

Sol: $R_1 = 80,$ $R_2 = ?$ $R_6 = 6\Omega.$

$$R_p = \frac{R_1 R_2}{R_1 + R_2} \Rightarrow 6 = \frac{8R_2}{8 + R_2}$$

$$\Rightarrow 48 + 6R_2 = 8R_2 \Rightarrow R_2 = 24\Omega$$

$$\text{From } R = \frac{\rho l}{A} \Rightarrow l = \frac{RA}{\rho}$$

(Data in sufficient. A is not given)

6. Three resistors $3\Omega, 6\Omega$ and 9Ω are connected to a battery. In which of them will the power dissipation be maximum if:

a) They all are connected in parallel b) They all are connected in series? Give reasons.

A. Given three resistors $R_1 = 3\Omega, R_2 = 6\Omega, R_3 = 9\Omega$

a) If they all are connected in parallel, potential difference across each of them remains same.

So effective resistance decreases

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_p} = \frac{1}{3} + \frac{1}{6} + \frac{1}{9}$$

$$\frac{1}{R_p} = \frac{6+3+9}{18}$$

$$R_p = \frac{18}{11} \Omega$$

So the power dissipation will be maximum in series combination ($\because P \propto R$)

7. A silver wire has a resistance of 2.1Ω at 27.5°C and a resistance of 2.7Ω at 100°C . determine the temperature coefficient of resistivity of silver?

$$\begin{aligned} \text{Sol: } \alpha_s &= \frac{R_2 - R_1}{R_1(t_2 - t_1)} = \frac{2.7 - 2.1}{2.1(62.5)} \\ &= \frac{0.6}{131.25} = 0.4 \times 10^{-2} \text{ } ^\circ\text{C} \end{aligned}$$

8. If the length of a wire conductor is doubled by stretching it while keeping the potential difference constant, by what factor will the drift speed of the electrons change?

A. Given the length of a wire in doubled by stretching.

If a wire is stretched, the volume of the conductor is constant.

$$R = \frac{\rho l}{A} = R = \frac{\rho l^2}{V} = \boxed{R \propto l^2}$$

We know $V = iR$

$$i = \frac{V}{R} = i \propto \frac{1}{R}$$

$$\text{i.e. } i \propto \frac{1}{l^2}$$

We know $i = neAV_d$

$$V_d = \frac{i}{neA} = V_d \propto \frac{i}{A}$$

$V_d \propto il$ (\because Volume is constant)

$$\frac{V_{d_2}}{V_{d_1}} = \frac{i_2}{i_1} \times \frac{l_2}{l_1} = \frac{V_{d_2}}{V_{d_1}} = \frac{i_1/4}{i_1} \times \frac{2l_1}{l_1}$$

$$\frac{V_{d_2}}{V_{d_1}} = \frac{1}{2} = V_{d_2} = \frac{V_{d_1}}{2}$$

9. Two 120V light bulbs, one of 25W and another of 200W are connected in series. One bulb burnt out almost instantaneously. Which one was burnt and why?

Sol: $p_1 = 25W, p_2 = 200W, V = 120V$

$$\text{From } p = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P}$$

$$R_1 = \frac{V^2}{P_1} = \frac{120 \times 120}{25} = 576\Omega$$

$$R_2 = \frac{V^2}{P_2} = \frac{120 \times 120}{200} = 72\Omega$$

When connected in series,

From $H = i^2 R t$ (i, t constant)

$$H \propto R$$

The 25w bulb having high resistance develops maximum heat and burnt out in instantaneously.

10. A cylindrical metallic wire is stretched to increase its length by 5%. Calculate the percentage change in resistance?

$$\text{Sol: } R \propto l^2 \Rightarrow \frac{\Delta R}{R} \times 100 = 2 \left(\frac{\Delta l}{l} \times 100 \right)$$

$$= 2(5) = 10\%$$

11. Two wires A and B of same length and same material, have their cross sectional areas in the ratio 1:4. What would they be ratio of heat produced in these wires is when the voltage across each is constant?

$$\text{Sol: } \frac{A_1}{A_2} = \frac{1}{4}, R = \frac{\rho l}{A}$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{A_2}{A_1} = \frac{4}{1} \text{ (s, l, constants)}$$

$$H = \frac{V^2}{R} t \Rightarrow \frac{H_1}{H_2} = \frac{R_2}{R_1} = \frac{1}{4}$$

12. Two bulbs whose resistances are in the ratio of 1:2 are connected in parallel to a source of constant voltage. What will be the ratio of power dissipation in these?

A. Given the ratio of resistance 1:2

$$R_1 : R_2 = 1 : 2$$

$$\text{Power dissipation } (P) = \frac{V^2}{R}$$

$$\text{i.e } P \propto \frac{1}{R} \quad [\because V \text{ is constant}]$$

$$\frac{P_1}{P_2} = \frac{R_2}{R_1} \Rightarrow \frac{P_1}{P_2} = \frac{2}{1} \Rightarrow P_1 : P_2 = 2 : 1$$

13. A potentiometer wire is 5 m long and a potential difference of 6 V is maintained between its ends. Find the emf of a cell which balances against a length of 180 cm of the potentiometer wire?

A. Given $\epsilon_1 = 6V$; $\epsilon_2 = ?$

$$l_1 = 5m; \quad l_2 = 180cm$$

$$\text{We know } \frac{\epsilon_1}{\epsilon_2} = \frac{l_1}{l_2}$$

$$\epsilon_2 = \epsilon_1 \times \frac{l_2}{l_1}$$

$$= 6 \times \frac{180 \times 10^{-2}}{5}$$

$$\epsilon_2 = 2.16V$$

14. A battery of emf 2.5 V and internal resistance r is connected in series with a resistor of 45 ohm through an ammeter of resistance 1 ohm. The ammeter reads a current of 50 mA. Draw the circuit diagram and calculate the value of r ?

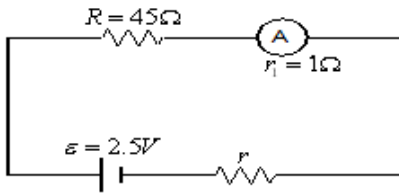
A. Battery of emf $\epsilon = 2.5V$

Internal resistance = r

External resistance (R) = 45Ω

Resistance of Ammeter (r_1) = 1Ω

The current in Ammeter (i) = 50mA



The current in the circuit $i = \frac{E}{R_s}$

$$i = \frac{E}{R + r_1 + r}$$

$$50 \times 10^{-3} = \frac{2.5}{45 + 1 + r}$$

$$50 \times 10^{-2} = \frac{2.5}{46 + r}$$

$$46 + r = \frac{2.5}{5 \times 10^{-2}} = 46 + r = 50$$

$$\therefore r = 4\Omega$$

The volume of internal resistance (r) = 4Ω

15. Amount of charge passing through the cross section of a wire is $q(t) = at^2 + bt + c$. Write the dimensional formula for a, b and c. If the values of a, b and c in SI unit are 6, 4, 2 respectively, find the value of current at $t=6$ seconds?

A: $q(t) = at^2 + bt + c$

$$a = 6, b = 4, c = 2$$

The value of current (i) at $t = 6\text{s} = ?$

$$i = \frac{dq}{dt} = \frac{d}{dt}(at^2 + bt + c)$$

$$i = 2at + b$$

$$i = 2 \times 6 \times 6 + 4$$

$$\therefore i = 76\text{amp}$$