ELECRTO STATIC POTENTIAL CAPACITANCE

Important Points:

1. Electric Potential:

Electric potential gives the electric condition of a body which determines the flow of charge. Electrons flow from a body at lower potential to a body at a higher potential.

2. Potential Difference:

Potential difference between two points in an electric field is defined as the work done in moving a unit positive charge from one point to another against the electric field. It is expressed in volt. It is a scalar quantity.

3. Equi-Potential Surface:

Equi -Potential Surface is the surface when the potential at all points are the surface is same. Work done to move a test charge on the Equi-Potential Surface is zero.

4. Capacitance:

The ability of a conductor to hold the charge is called the capacity of a conductor.

Capacitance of a conductor $C = \frac{Q}{V}$

5. Capacity of a parallel plate capacitor $C = \frac{\in_0 A}{d}$

6. If the space between the plates of the condenser is filled with a dielectric of constant K, then $C^1 = \frac{K \in_0 A}{d}$

7. Series Combination:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

The reciprocal of equivalent capacity is the sum of the reciprocals of the individual capacities

8. Parallel Combination:

$C = C_1 + C_2 + C_3$

The effective capacity is equal to the sum of individual capacities.

9. Energy stored in a condenser $U_0 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$

10. Effect of Dielectric in a Capacitor:

A dielectric is a solid insulating medium like glass, ebonite, wax paper, mica placed in between the two metal sheets in order to increase its capacity. These do not allow the electric charges to easily pass through them.

11. Dielectrics:

Dielectric is a material in which all the electrons are tightly bound to the nuclei of the atoms. Thus there are no free electrons to carry the current. The electrical conductivity of a dielectric is zero. The molecules of dielectric may be classified as non polar and polar.

12. A non-polar molecule is one in which the centre of gravity of positive charges coinciding with the centre of gravity of negative charges. A non-polar molecule has zero electric dipole moment in the absence of the external electric field.

13. A polar molecule is one in which the centre of gravity of positive and negative charges do not coincide. They are separated by finite distances. (When such polar molecules are placed in an external electric field, the field tries to orient the positive charge centers in the direction of the field and negative charge centers in the opposite direction) The polar molecule is an electric dipole and has a permanent dipole moment.

14. Van de Graff Generator:

i) The electric charges take place in air or gases readily at pointed Conductors.

ii) If a hollow conductor is in contact with another conductor, then as charge is supplied to the conductor, the charge shifts immediately to outer surface of the hollow conductor.

Very Short Answer Questions

1. Can there be electric potential at a point with zero electric intensity? Give an example?

A. Yes. There can be electric potential with zero electric intensity.

Ex: Inside a charged spherical conductor, electric intensity is zero but electric potential is not zero.

2. Can there be electric intensity at a point with zero electric potential? Give an example?

A. Yes. There can be electric intensity with zero electric potential.

Ex. When two dissimilar charges of same magnitude are separated by a certain distance at the midpoint potential is zero. But electric field strength is not zero.

3. What are meant by equipotential surfaces?

A. Equi-Potential Surface:

It is a surface in an electric field on which the potential is same at every point. Or the locus of all points which have the same electric potential is called equipotential surface. Work done to move a test charge from one point to another on Equi-Potential Surface is zero.

4. Why is the electric field always at right angles to the equipotential surface? Explain?

A. If the electric field is not at right angles to the equipotential surface, it would have a non zero component along the surface. Hence work has to be done to move a test charge against this component. This is against to the definition and hence the electric field always at right angles to the Equi-Potential Surface.

5. Three capacitors of capacitances $1\mu F, 2\mu F$ *and* $3\mu F$ are connected in parallel.

a) What is the ratio of charges b) What is the ratio of potential differences?

A. **a)** In parallel combination, $Q \propto C$

$\therefore Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3 = 1:2:3$

b) As the potential is same on all capacitors $V_1 : V_2 : V_3 = 1:1:1$

6. Three capacitors of capacitances $1\mu F$, $2\mu F$ and $3\mu F$ are connected in series

a) What is the ratio of charges?

b) What is the ratio of potential differences?

A: In series combination, charge is same on each capacitor

Ratio of charges = $Q_1 : Q_2 : Q_3$ = 1 : 1 : 1

And $V \propto \frac{1}{C}$

Ratio of potential differences = $v_1 : v_2 : v_3 = \frac{q}{C_1} : \frac{q}{C_2} : \frac{q}{C_3} = \frac{1}{1} : \frac{1}{2} : \frac{1}{3} = 6 : 3 : 2$

7. What happens to the capacitance of a parallel plate capacitor if the area of the plates is doubled?

A: The capacitance of a parallel plate capacitor

c = $\frac{\epsilon_o A}{d} \Rightarrow c \alpha A$

If area doubled capacitance also doubled

8. The dielectric strength of air is 3×10^6 V/m at certain pressure. A parallel plate capacitor with air in between the plates has a plate separation of 1 cm can you charge the capacitor to 3×10^6 V?

A: Distance between the plates $d = 1\ cm = 1\times10^{-2} m$

We know that strength of electric field (E) = $\frac{Potential(V)}{Distance(d)}$

$$E = \frac{3\times10^{6}}{10^{-2}} = 3\times10^{8}\ V/m$$

Which is more than dielectric strength of air so it not possible

Short Answer Questions

1. Derive an expression for the electric potential due a point charge?

A. **Electric Potential:**

The work done to bring a unit positive charge from infinity to that point in the electric field is called the electric potential at that point.

$$V = \frac{1}{4\pi\epsilon_o}\frac{q}{r}$$

Explanation:

Consider a positive charge 'Q' be placed at a point O in free space. Let P be a point at a distance r from O. Let E be the intensity of the electric field at P.

Work done in bringing a unit positive test charge from infinity to the point P, against the repulsive force of charge Q (Q > 0), is the potential at P due to the charge Q.

Let a unit positive charge is moved from r^1 to $\left(r+\Delta r^1\right)$. The work done in doing so is given by

$$\Delta W = -\frac{Q}{4\pi\epsilon_o r^{1^2}}\Delta r^1$$

$dW = -Edx.$ The negative sign indicates that E and dx are opposite.

The total work done in moving the unit positive charge from infinity to a point $r^1 = r$ from 'q' is given by-

$$W = \int_{\infty}^{r} \frac{Q}{4\pi\epsilon_o r^{1^2}} \; dr^1 = \frac{Q}{4\pi\epsilon_o r^1} \int_{\infty}^{r} = \frac{Q}{4\pi\epsilon_o r}$$

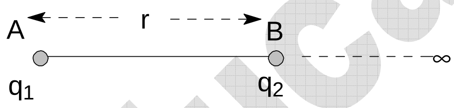
This work done gives the potential at P due to the charge Q.

$$\therefore V = \frac{1}{4\pi\epsilon_o} \frac{Q}{r}$$

2. Derive an expression for the electrostatic potential energy of a system of two point charges and find its relation with electric potential of a charge?

A. Work done in bringing the given charge from infinity to a point in the electric field is known as potential energy of the charge.

Let us consider a positive charge q_1 is at A, whose Potential at B, $V = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r}$



In bringing another positive charge q_2 from infinite to a point B

Work done is $W = Vq_2 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$

This amount of work done stores as potential energy of system of charges

$$\therefore P.E(U) = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

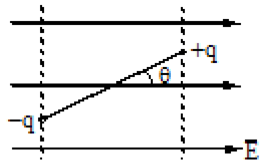
Work done in bringing the unit charge from infinity to a point in the electric field is known as potential at a point.

Work done in bringing the given charge from infinity to a point in the electric field is known as potential energy of the charge.

So potential energy = potential at a point (v) x charge (q).

3. Derive an expression for the potential energy of an electric dipole placed in a uniform electric field?

A. Consider an electric dipole of length 2*l* and dipole moment P in a uniform electric field as shown. Let θ be the angle between dipole moment and field direction.



The work done by an external agent to rotate the dipole through a given angle in the field is then stored as potential energy in the system of the dipole and the external field. The work dW required to rotate the dipole through an angle is given by

$$dW = \tau\, d\theta$$

But, $\tau = pE\sin\theta$

This work is transformed into potential energy U. We find this for a rotation from θ_0 to θ. So,

$$U = \int_{\theta_0}^{\theta} \tau\, d\theta = \int_{\theta_0}^{\theta} pE\sin\theta\, d\theta = pE\int_{\theta_0}^{\theta} \sin\theta d\theta$$

$$\Rightarrow U = pE\left(-\cos\theta\right)\Big|_{\theta_0}^{\theta} = pE\left(\cos\theta_0 - \cos\theta\right)$$

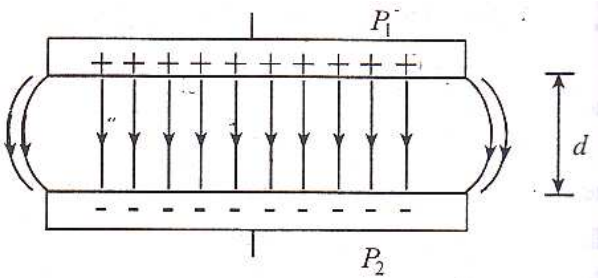
The term involving $\cos\theta_0$ is a constant that depends on the initial orientation of the dipole.

4. Derive an expression for the capacitance of a parallel plate capacitor?

A. A parallel plate capacitor consist of two conducting metal plates P_1 and P_2 each of area A, separated by a distance d. If we connect a battery of +ve terminal to P_1 and negative terminal to P_2. The charge is uniformly distributed to the plates. Hence electric field will be uniform between two plates.

At the edges field will not uniform. This is called fringing field and its effect can be neglected if d<<A.

Electric field intensity at any point between the plates



We know that

$$\therefore E = \frac{\sigma}{\epsilon_0} \text{ (Air between the plates)}$$

$$\Rightarrow E = \frac{q}{A\epsilon_0} \left(\because \sigma = \frac{q}{A}\right)$$

The P.D between the plates V = Ed

$$V = \left(\frac{q}{A\epsilon_0}\right)d$$

Capacity of air filled parallel plate capacitor $C_0 = \frac{q}{V}$

$$C_0 = \frac{A\epsilon_0}{d}$$

If the dielectric material is placed between the plates the capacitance of parallel plate capacitor

$$C = \frac{KA\epsilon_0}{d} \left(\because K = \frac{\epsilon_m}{\epsilon_0} \Rightarrow \epsilon_m = K\epsilon_0\right)$$

5. Explain the behavior of a dielectric in an external field?

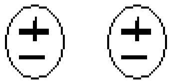
A. Dielectrics:

Substances which do not allow the electric charges to easily pass through them are called dielectrics or insulators.

Non-Polar Dielectrics:

Dielectric is a material in which all the electrons are tightly bound to the nuclei of the atoms. Thus there are no free electrons to carry the current. The electrical conductivity of a dielectric is zero. The molecules of dielectric may be classified as non polar and polar.

A non-polar molecule is one in which the centre of gravity of positive charges coinciding with the centre of gravity of negative charges. A non-polar molecule has zero electric dipole moment in the absence of the external electric field.



Ex: $H_2, N_2, O_2, CO_2 CH_4, CCl_4$ etc.

Polar Dielectrics:

A polar molecule is one in which the centre of gravity of positive and negative charges do not coincide. They are separated by finite distances. (When such polar molecules are placed in an external electric field, the field tries to orient the positive charge centers in the direction of the field and negative charge centers in the opposite direction) The polar molecule is an electric dipole and has a permanent dipole moment.

Ex: HCL, CO, H_2O, NH_3 etc.

Electric Polarization of Dielectric:

If a dielectric slab of dielectric material is placed in a uniform electric field E_o between the parallel plates of a capacitor. Then the slab becomes electrically polarised. These induced charges produce an electric field (E_P) inside the dielectric the opposite direction to that of external field. Hence the magnitude of the resultant field intensity within the dielectric is given by

$$|\overline{E}| = |\overline{E_o}| - |\overline{E_p}| \ (or) \ E = E_0 - E_p$$

Long Answer Questions

1. Define electric potential. Derive an expression for the electric potential due to an electric dipole and hence the electric potential at a point (a) on the axial line of electric dipole (b) on the equatorial line of electric dipole.

A. Electric Potential (V):

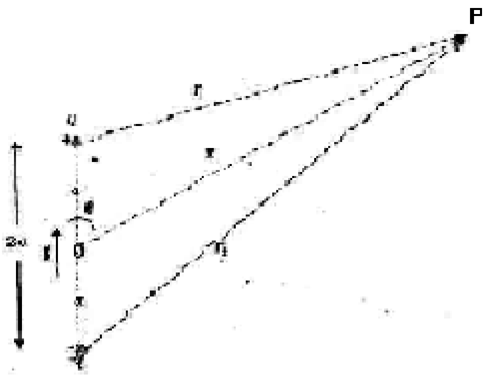
The work done to bring a unit positive charge from infinity to that point in the electric field is called the electric potential at that point.

$V = \frac{1}{4\pi \in_0} \frac{q}{d}$ This is a scalar quantity.

Potential due to a Dipole:

Consider a dipole consisting of charge ‘q’ and length 2a.The electric dipole moment is given by

P = (2a) q.Let the origin is at the centre of the dipole. The potential due to the dipole is the sum of potentials due to the charges +q and -q .



$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right)$ Where r_1 and r_2 are the distances of the point P from +q and -q respectively.

But $r_1^2 = r^2 + a^2 - 2ar\cos\theta \cong r^2\left(1 - \frac{2a\cos\theta}{r}\right)$ and $r_1^2 = r^2 + a^2 + 2ar\cos\theta \cong r^2\left(1 + \frac{2a\cos\theta}{r}\right)$

$\frac{1}{r_1} \cong \frac{1}{r}\left(1 - \frac{2a\cos\theta}{r}\right)^{-1/2} \cong \frac{1}{r}\left(1 + \frac{a\cos\theta}{r}\right)$ And $\frac{1}{r_2} \cong \frac{1}{r}\left(1 + \frac{2a\cos\theta}{r}\right)^{-1/2} \cong \frac{1}{r}\left(1 - \frac{a\cos\theta}{r}\right)$

$$V = \frac{q}{4\pi\varepsilon_0}\frac{2a\cos\theta}{r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

Hence potential varies inversely as the square of the distance from the dipole.

Axial Line:

For a point on the axial line $\theta = 0$

$$\therefore V_{\text{axial}} = P / 4\pi\varepsilon_0 r^2$$

Equatorial Line:

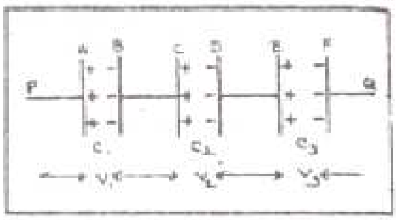
For a point on the equatorial line $\theta = 90^0$

$$\therefore V_{\text{equitorial}} = 0$$

2. Explain series and parallel combination of capacitors. Derive the formula for equivalent capacitance in each combination?

A. Series Combination:

Let three capacitors of capacities C_1 C_2 and C_3 are connected in series. Let V be the potential difference across the combination. Let V_1 V_2 and V_3 be the potential difference across the capacitors respectively. As the capacitors are in series the charge an all capacitors is same and let it be q.



Series Combination

$$V_1 = \frac{q}{C_1}; V_2 = \frac{q}{C_2} \text{ And } V_3 = \frac{q}{C_3}$$

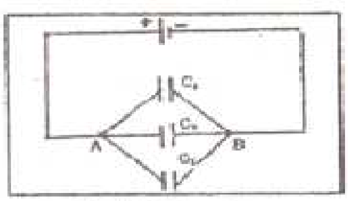
But $V = V_1 + V_2 + V_3$

$$\therefore \frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} \qquad \text{(Or)} \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Thus the reciprocal of equivalent capacity is the sum of the reciprocals of the individual capacities.

Parallel Combination:

Let three capacitors of capacities C_1 C_2 and C_3 are connected in series. Let V be the potential difference across the combination .Let q_1 q_2 and q_3 be the charges on the capacitors respectively. As the capacitors are connected parallel the potential difference across each capacitor is same and let it be V.



Parallel Combination

$q_1 = C_1V;\ q_2 = C_2V\ \ and\ \ q_3 = C_3V$

But $q = q_1 + q_2 + q_3$

$CV = C_1V + C_2V + C_3V$ (Or) $\therefore C = C_1 + C_2 + C_3$

Thus the effective capacity is equal to the sum of individual capacities.

3. **Derive an expression for the energy stored in a capacitor. What is the energy stored when the space between the plates is filled with a dielectric?**

a) With charging battery disconnected?

b) With charging battery connected in the circuit?

A. Energy stored in a Capacitor:

Consider a capacitor of capacity C which is charged to a potential V. Let 'q' be the charge on it.

$\therefore V = \dfrac{q}{C}$

The work done in increasing the charge on the condenser by a small amount dq is given by

$$dW = V\,dq = \frac{q}{c}dq$$

The total work done in increasing the charge on the condenser from 0 to q is given by,

$$W = \frac{1}{C}\int_0^q q\,dq = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C}$$

This work done is stored as electric potential energy of the condenser.

$$\therefore U_0 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Effect of dielectric on Energy:

Let the C_0 be the capacity and V_0 be the potential of a capacitor with air as dielectric. Let q_0 be the charge on the capacitor. The energy stored in the condenser is $U_o = \frac{1}{2}q_o V_o$

a) When The Charging Battery Is Disconnected:

If the battery is disconnected from the capacitor after charging it and the space between the plates is filled completely with a dielectric of constant k, its potential becomes $V = \frac{V_0}{k}$.

Since the battery is disconnected the charge on it is constant. Hence energy stored is

$$U = \frac{1}{2}qV = \frac{1}{2}q_0\frac{V_0}{k} = \frac{1}{2}\frac{q_0V_0}{k}$$

$$\therefore U = \frac{U_0}{k}$$

Hence energy reduces by a factor k.

b) When The Charging Battery Connected.

Let the dielectric of constant k is introduced between the plates of the condenser keeping the battery connected to the capacitor. Hence the charge on the plates increases to $q = kq_0$

Since the battery is not disconnected, the potential difference remains the same. The energy stored is given by $U = \frac{1}{2}qV = \frac{1}{2}(kq_0)V_0 = kU_0 \Rightarrow U = kU_0$

Hence the energy increases by a factor k.

PROBLEMS

1. An elementary particle of mass 'm' and charge + e initially at a very larger distance is projected with velocity 'v' at a much more massive particle of charge + Ze at rest. The closet possible distance of approach of the incident particle is.

Sol: $q_1 = e, \quad q_2 = Ze.$

$\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = \frac{1}{2}mv^2$ Where 'r' closest distance of approach

$$\frac{1}{4\pi\varepsilon_0}\frac{(e)(ze)}{r} = \frac{1}{2}mv^2$$

$$\Rightarrow r = \frac{2ze^2}{4\pi\varepsilon_0 \; mv^2} = \frac{Ze^2}{2\pi\varepsilon_0 \; mv^2}$$

2. In a Hydrogen atom the electron and proton are at a distance of 0.5 A^0. The dipole moment of the system is

Sol: $2l = 0.5A^o = 5\times 10^{-11} m$

$$q = 1.6\times 10^{-19} C$$

$$p = 2lq = 5\times 10^{-11}\times 1.6\times 10^{-19}$$

$$= 8\times 10^{-30} cm$$

3. There is a uniform electric field in the XOY plane represented by $\left(40\hat{i} + 30\hat{j}\right)Vm^{-1}$. If the electric potential at the origin is 200 V, the electric potential at the point with coordinates (*2m*, 1*m*) is

Sol: $\vec{r} = \left(2\hat{i} + 1\hat{j}\right)m$ and $\vec{E} = \left(40\hat{i} + 30\hat{j}\right)vm^{-1}$

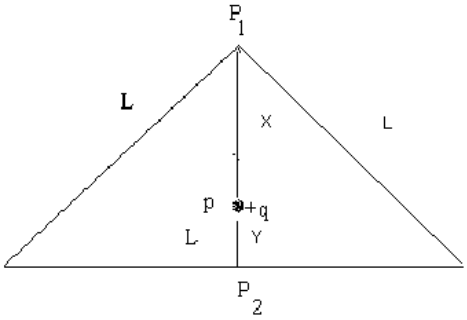
Potential at origin = v_O = 200 v

$v_p - v_o = -\vec{E}.\vec{r} = -$ (40i +30 j). (2i + j)

$$v_p - 200 = -110 \Rightarrow v_p = 200 - 110 = 90v$$

4. An equilateral triangle has a side length L, a charge +q is kept at the centroid of the triangle P is a point on the perimeter of the triangle. Find the ratio of the minimum and maximum possible electric potentials for the point P?

A.



Equilateral triangle of side length L from the figure x = $\frac{L}{\sqrt{3}}$, $Y = \frac{L}{2\sqrt{3}}$

At point P_1 potential is minimum because distance is maximum.

$$V_{min} = V_{P_1} = \frac{q}{4\pi\in_0 \frac{L}{\sqrt{3}}} = \frac{\sqrt{3}q}{4\pi\in_0 L}$$

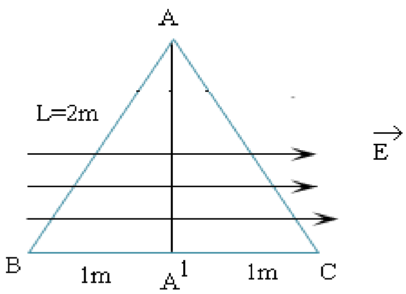
At point P_2 potential is maximum because distance is minimum

$$V_{max} = V_{P_2} = \frac{q}{4\pi\in_0\left(\frac{L}{2\sqrt{3}}\right)} = \frac{2\sqrt{3}q}{4\pi\in_0 L}$$

$$\therefore \frac{V_{min}}{V_{max}} = \frac{1}{2}$$

5. ABC is an equilateral triangle of side 2*m*. There is a uniform electric field of intensity 100 V/m in the plane of the triangle and parallel to BC as shown. If the electric potential at A is 200 V, then the electric potentials at B and C are respectively?

A. The uniform electric field of intensity = 100 V/m



The electripotential at point A = 200 V

$AA^1 \rightarrow$ is equatorial line so $V_A = V_{A^1}$

We know that $dv = E\, dr$

$V_B - V_{A^1} = 100 \times 1$

$V_B 200 = 100$

$V_B = 300$ volt

$V_C = 100$

$V_C = V_{A^1} - 100 = 200 - 100$

$V_C = 100$ volts

6. An electric dipole of moment P is placed in a uniform electric field E. with P parallel to E. It is then rotated by an angle. The work done is

Sol: work done is rotating dipole angle $'d\theta'$

$$(dW) = \tau\, d\theta$$

$$\Rightarrow dW = PE \sin\theta\, d\theta$$

Work done in rotating dipole from angle $\theta = \theta_1$ to $\theta = \theta_2$

Is given by $W = \int_{\theta_1}^{\theta_2} dw$

$$\Rightarrow W = \int_{\theta_1}^{\theta_2} PE \sin\theta d\theta = PE \int_{\theta_1}^{\theta_2} \sin\theta d\theta$$

$$\Rightarrow W = PE(-\cos\theta)_{\theta_1}^{\theta_2}$$

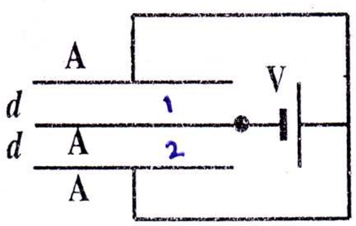
$$\Rightarrow W = PE(\cos\theta_1 - \cos\theta_2)$$

If $\theta_1 = o^o$, $\theta_2 = Q$

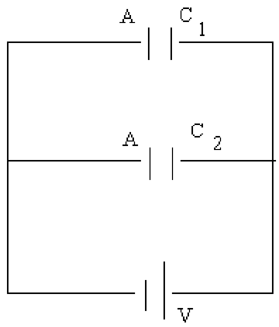
$$\Rightarrow W = PE(\cos o^o - \cos Q)$$

$$W = PE(1 - \cos Q)$$

7. Three identical metal plates each of area 'A' are arranged parallel to each other. 'd' is the distance between the plates as shown. A battery 'V' volts is connected as shown. The charge stored in the system of plates is



A: The equivalent circuit of above circuit is



The area of each capacitor is A

The distance between the plates is d

Capacitance of first capacitor $C_1 = \frac{\in_0 A}{d}$

Capacitance of second capacitor $C_2 = \frac{\in_0 A}{d}$

Resultant capacitance $C = C_1 + C_2$

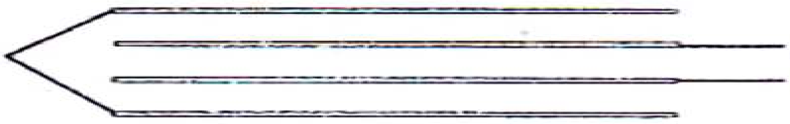
$C = \frac{\in_0 A}{d} + \frac{\in_0 A}{d}$

$C = \frac{2\in_0 A}{d}$

The charge stored in the system of plates is $Q = CV$

$Q = \frac{2\in_0 A}{d}V$

8. Four identical metal plates each of area a are separated mutually by a distance d and are connected as shown. Find the capacity of the system between the terminals A and B.



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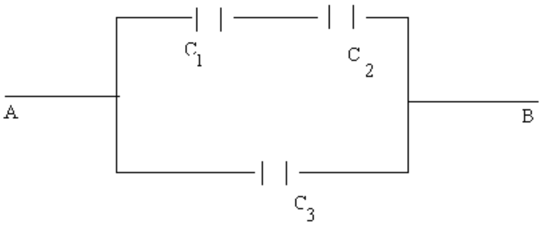


The capacitance of First Capacitor $C_1 = \frac{\epsilon_0 A}{d}$

The capacitance of 2nd capacitor $C_2 = \frac{\epsilon_0 A}{d}$

The capacitance of 3rd capacitor $C_3 = \frac{\epsilon_0 A}{d}$

The resultant capacitance of one and two is

$$\frac{1}{C^1} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\frac{\epsilon_0 A}{d}} + \frac{1}{\frac{\epsilon_0 A}{d}} = \frac{2d}{\epsilon_0 A}$$

$$C^1 = \frac{\epsilon_0 A}{2d}$$

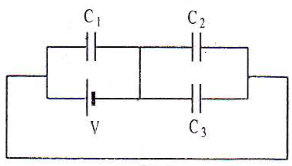
C^1 is parallel to C_3

$\therefore$ The capacity of the system is

$$C = C^1 + C_3 = \frac{\epsilon_0 A}{2d} + \frac{\epsilon_0 A}{d}$$

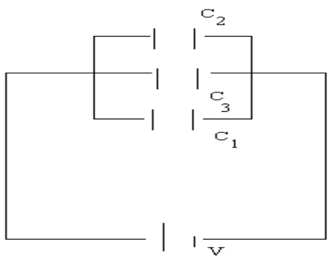
$$C = \frac{3}{2}\frac{\epsilon_0 A}{d}$$

9. In the circuit shown the battery of 'V' volts has no internal resistance. All three condensers are equal in capacity. Find the condenser that carries more charge?



A: The equivalent circuit of above circuit is given that all the three condensers have equal capacity

Three condensers are parallel to each other so potential also same.



$C = \frac{q}{V} \Rightarrow q = CV$

Hence all three carry same charge

10. Two capacitors A and B of capacities C and 2C are connected in parallel and the combination is connected to a battery of V volts. After the charging is over, the battery is removed. Now a dielectric slab of K = 2 is inserted between the plates of A so as to fill the space completely. The energy lost by the system during the sharing of charges is

A: When the two capacitors connected to a battery of V volts

The equivalent capacitance $C + 2C = 3C$

The energy stored is $U_i = \frac{1}{2}(3c)V^2$

$U_i = \frac{3}{2}cV^2$

When the battery is disconnected and a dielectric of constant K = 2 inserted between plates of A

The charge on the capacitors system is remains same as that of earlier.

$Q = C_{eff}V$

$Q = 3CV$

Now the charge on each capacitor is $q = \frac{3cv}{2}$

The capacitance of capacitor A is $C^1 = KC = 2C$

The capacitance of capacitor B is 2C

Now new energy of system $U_f = U_A + U_B$

$$U_f = \frac{Q^2}{2C_A} + \frac{Q^2}{2C_B}$$

$$U_f = \frac{\left(\frac{3}{2}CV\right)^2}{2(2C)} + \frac{\left(\frac{3}{2}CV\right)^2}{2(2C)}$$

$$U_f = \frac{9CV^2}{8}$$

Energy lost by the system during the sharing of charges is

$$U_{lost} = U_i - U_f$$

$$= \frac{3}{2}CV^2 - \frac{9}{8}CV^2$$

$$U_{lost} = \frac{3}{8}CV^2$$

11. A condenser of certain capacity is charged to a potential V and stores some energy. A second condenser of twice the capacity is to store half the energy of the first, find to what potential one must be charged?

Sol: $C_2 = 2c_1,\ U_2 = \frac{1}{2}U_1$

$$U = \frac{1}{2}cV^2 \Rightarrow V^2 \propto \frac{U}{C}$$

$$\left(\frac{V_2}{V_1}\right)^2 = \left(\frac{U_2}{U_1}\right)\left(\frac{C_1}{C_2}\right) \Rightarrow \left(\frac{V_2}{v}\right)^2 = \left(\frac{1}{2}\frac{U_1}{V_1}\right)\left(\frac{C_1}{2C_1}\right) = \frac{1}{4}$$

$$\frac{V_2}{V} = \frac{1}{2} \Rightarrow V_2 = \frac{V}{2}$$