

Total No. of Questions : 24
Total No. of Printed Pages : 4

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Part-III

MATHEMATICS, Paper - I (A)

(English version)

Time : 3 Hours]

[Max. Marks : 75

Note : This question paper consists of **three** sections **A, B** and **C**.

SECTION - A

10×2=20

I. Very short answer type questions.

- (i) Answer **All** the questions.
(ii) Each question carries **TWO** marks.

1. If $f: \mathbb{R} \rightarrow \mathbb{R}$, $g: \mathbb{R} \rightarrow \mathbb{R}$ are defined by $f(x) = 3x - 1$, $g(x) = x^2 + 1$, then find $(f \circ g)(2)$.

2. Find the domain of the function $f(x) = \frac{1}{\sqrt{1-x^2}}$, where f is real valued function.

3. Define Trace of the Matrix and

find the trace of the matrix $A = \begin{bmatrix} 1 & 3 & -5 \\ 2 & -1 & 5 \\ 2 & 0 & 1 \end{bmatrix}$

4. If $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & -2 \\ -1 & x & 0 \end{bmatrix}$ is a Skew symmetric matrix, then find x .
5. Find the vector equation of the Plane passing through the points $\bar{i} - 2\bar{j} + 5\bar{k}$, $-5\bar{j} - \bar{k}$ and $-3\bar{i} + 5\bar{j}$.
6. Find the unit vector in the direction of the sum of the vectors $\bar{a} = 2\bar{i} + 2\bar{j} - 5\bar{k}$ and $\bar{b} = 2\bar{i} + \bar{j} + 3\bar{k}$.
7. Let $\bar{a} = \bar{i} + \bar{j} + \bar{k}$ and $\bar{b} = 2\bar{i} + 3\bar{j} + \bar{k}$, find projection vector of \bar{b} on \bar{a} and its magnitude.
8. If $\sin \alpha + \operatorname{cosec} \alpha = 2$, find the value of $\sin^n \alpha + \operatorname{cosec}^n \alpha$, $n \in \mathbb{Z}$.
9. Find a sine function, whose period is $\frac{2}{3}$.
10. If $\cosh x = \sec \theta$, then prove that $\tanh^2 \frac{x}{2} = \tan^2 \frac{\theta}{2}$.

SECTION - B

5×4=20

II. Short answer type questions.

- (i) Answer **ANY FIVE** questions.
- (ii) Each question carries **FOUR** marks.

11. Show that $\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ a & b & c \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$.

12. Find the point of intersection of the line $\bar{r} = 2\bar{a} + \bar{b} + t(\bar{b} - \bar{c})$ and the plane $\bar{r} = \bar{a} + x(\bar{b} + \bar{c}) + y(\bar{a} + 2\bar{b} - \bar{c})$, where \bar{a} , \bar{b} and \bar{c} are non-coplanar vectors.
13. Prove that angle in a semi-circle is a right angle by using Vector method.
14. Prove that $\sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{5\pi}{8} + \sin^4 \frac{7\pi}{8} = \frac{3}{2}$.
15. Solve the equation $2 \cos^2 \theta + 11 \sin \theta = 7$ and write general solution.
16. Prove that $\cos^{-1} \left(\frac{4}{5} \right) + \sin^{-1} \left(\frac{3}{\sqrt{34}} \right) = \tan^{-1} \left(\frac{27}{11} \right)$.
17. In a ΔABC , if $a : b : c = 7 : 8 : 9$, then find $\cos A : \cos B : \cos C$.

SECTION - C

5×7=35

III. Long answer type questions.

(i) Answer **ANY FIVE** questions.

(ii) Each question carries **SEVEN** marks.

18. Let $f : A \rightarrow B$, $g : B \rightarrow C$ are bijections, then prove that $g \circ f : A \rightarrow C$ is a bijection.

19. By Mathematical Induction, show that $49^n + 16n - 1$ is divisible by 64 for all positive Integer n .

20. If $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is a non-singular matrix,

then show that A is invertible and $A^{-1} = \frac{\text{Adj } A}{\det A}$.

21. Solve the following equations by using Cramer's rule -

$$2x - y + 3z = 9$$

$$x + y + z = 6$$

$$x - y + z = 2.$$

22. If $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}$, $\vec{b} = 2\vec{i} + \vec{j} + \vec{k}$, $\vec{c} = \vec{i} + \vec{j} + 2\vec{k}$,

then find $|(\vec{a} \times \vec{b}) \times \vec{c}|$ and $|\vec{a} \times (\vec{b} \times \vec{c})|$.

23. In triangle ABC , prove that

$$\cos \frac{A}{2} + \cos \frac{B}{2} + \cos \frac{C}{2} = 4 \cos \left(\frac{\pi - A}{4} \right) \cos \left(\frac{\pi - B}{4} \right) \cos \left(\frac{\pi - C}{4} \right)$$

24. In ΔABC , if $r_1 = 8$, $r_2 = 12$, $r_3 = 24$; find a , b and c .
