193

(New Syllabus)

III

Total No. of Questions - 24

Total No. of Printed Pages - 3

Regd. No.

Part - III

MATHEMATICS, Paper - I (B)

(Co-ordinate Geometry and Calculus)

(English Version)

Time : 3 hours *Max. Marks : 75*

Note : This question paper consists of **three** sections *A*, *B* and *C*.

SECTION A

I. **Very short** answer type questions. 10 × 2 = 20

i) Attempt **all** questions.

ii) Each question carries **two** marks.

1. Find the equation of the straight line passing through (–2, 4) and making nonzero intercepts whose sum is zero.

2. Find the distance between the parallel straight lines $3x+4y-3=0$ and $6x+8y-1=0$.

3. Reduce the equation $x+2y-3z-6=0$ of the plane to the normal form.

4. Compute $\underset{x \to 0}{Lim} \dfrac{e^{7x}-1}{x}$.

5. Show that $\underset{x \to \infty}{Lim} \left(\sqrt{x+1}-\sqrt{x}\right)=0$.

6. If $(3, 2, -1)$, $(4, 1, 1)$ and $(6, 2, 5)$ are three vertices and $(4, 2, 2)$ is the centroid of a tetrahedron, find the fourth vertex.

7. If $y=\left(Cot^{-1}x^3\right)^2$, then find $\dfrac{dy}{dx}$.

8. If $y = e^{2x} \cdot Log(3x+4)\left(x > \frac{-4}{3}\right)$, then find $\frac{dy}{dx}$.

9. If $y = e^x + x$, $x = 5$, $\Delta x = 0.02$, then find Δy and dy.

10. Find the value of 'c' in Rolle's theorem for the function $f(x) = x^2 - 1$ on $[-1, 1]$.

SECTION B

II. Short answer type questions. **5 × 4 = 20**

i) Attempt **any five** questions.

ii) Each question carries **four** marks.

11. Find the equation of the locus of P, if $A = (4, 0)$, $B = (-4, 0)$ and $|PA - PB| = 4$.

12. When the axes are rotated through an angle $\frac{\pi}{6}$, find the transformed equation of $x^2 + 2\sqrt{3}xy - y^2 = 2a^2$.

13. If p and q are the lengths of the perpendiculars from the origin to the straight lines $x\,Sec\,\alpha + y\,Cosec\,\alpha = a$ and $x\,Cos\,\alpha - y\,Sin\,\alpha = a\,Cos\,2\alpha$, prove that $4p^2 + q^2 = a^2$.

14. Find the real constants a, b so that the function f given by

$$f(x) = \begin{cases} Sin\,x & \text{if } x \le 0 \\ x^2 + a & \text{if } 0 < x < 1 \\ bx + 3 & \text{if } 1 \le x \le 3 \\ -3 & \text{if } x > 3 \end{cases}$$

is continuous on R.

15. Find the derivative of $Sin\,2x$ from the first principle.

16. A container is in the shape of an inverted cone has a height 8 m and radius 6 m at the top. If it is filled with water at the rate of 2 m^3/minute, how fast is the height of water changing when the level is 4 m?

17. Find the equations of the tangent and the normal to the curve $y^4 = ax^3$ at (a, a).

SECTION C

III. Long answer type questions. 5 × 7 = 35

i) Attempt **any five** questions.

ii) Each question carries **seven** marks.

18. Find the orthocenter of the triangle whose sides are given by $x+y+10=0$, $x-y-2=0$ and $2x+y-7=0$.

19. If the equation $ax^2+2hxy+by^2=0$ represents a pair of intersecting lines, then prove that the combined equation of the pair of bisectors of the angles between these lines is $h\left(x^2-y^2\right)=(a-b)xy$.

20. Find the condition for the chord $lx+my=1$ of the circle $x^2+y^2=a^2$ (whose center is the origin) to subtend a right angle at the origin.

21. Find the angle between the diagonals of a cube.

22. If $y=x^{Tan\,x}+(Sin\,x)^{Cos\,x}$, find $\frac{dy}{dx}$.

23. Find the angle between the curves $xy=2$ and $x^2+4y=0$.

24. If the curved surface of a right circular cylinder inscribed in a sphere of radius R is maximum, show that the height of the cylinder is $\sqrt{2}R$.
