

Total No. of Questions – 24

Regd.

Total No. of Printed Pages - 3

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Part - III
MATHEMATICS, Paper – II (A)
(English Version)

Time : 3 Hours]

[Max. Marks : 75

Note : This question paper consists of **three** Sections A, B and C.**SECTION – A****10 × 2 = 20**

I. Very Short Answer Type questions :

(i) Attempt **all** questions.(ii) Each question carries **two** marks.1. If α and β are the roots of $ax^2 + bx + c = 0$, find the value of $\alpha^2 + \beta^2$.2. If the product of the roots of $4x^3 + 16x^2 - 9x - a = 0$ is 9, then find a.

3. Define symmetric matrix and give an example.

4. If $A = \begin{pmatrix} 1 & 5 & 3 \\ 2 & 4 & 0 \\ 3 & -1 & -5 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & 0 \\ 0 & -2 & 5 \\ 1 & 2 & 0 \end{pmatrix}$, then find $3A - 4B'$.

5. Find the number of ways of arranging the letters of the word INDEPENDENCE.

6. Find the number of positive divisors of 1080.

7. Write down and simplify 6th term in $\left(\frac{2x}{3} + \frac{3y}{2}\right)^9$.8. Find the coefficient of x^k in the series expansion of $\frac{1 - ax - x^2}{e^x}$.

9. Define mutually exclusive events and give an example.

10. A Poisson variable satisfies $P(X = 1) = P(X = 2)$. Find $P(X = 5)$.

II. Short Answer Type questions :

- (i) Attempt any **five** questions.
 (ii) Each question carries **four** marks.

11. Determine the range of the expression $\frac{x+2}{2x^2+3x+6}$.

12. If $A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$ is a non-singular matrix, then show that A is invertible

and $A^{-1} = \frac{\text{Adj } A}{\det A}$.

13. If the letters of the word PRISON are permuted in all possible ways and the words thus formed are arranged in dictionary order, find the rank of the word PRISON.

14. Find the number of ways of forming a committee of 5 persons from a group of 5 Indians and 4 Russians such that there are at least 3 Indians in the committee.

15. Resolve $\frac{1}{(x-1)^2 \cdot (x-2)}$ into partial fractions.

16. Show that $\frac{1}{2x+1} + \frac{1}{3 \cdot (2x+1)^3} + \frac{1}{5(2x+1)^5} + \dots = \log_e \sqrt{\frac{x+1}{x}}$.

17. If one card is drawn from a pack of cards, then show that the event of getting an ace and getting a heart are independent events.

III. Long Answer Type questions :

- (i) Attempt any **five** questions.
 (ii) Each question carries **seven** marks.

18. Solve the equation : $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$.

19. Show that $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a - b)(b - c)(c - a)(ab + bc + ca)$.

20. Solve the system of equations : $2x - y + 3z = 9$, $x + y + z = 6$, $x - y + z = 2$, by Gauss-Jordan method.

21. Find the sum to infinite terms of the series

$$\frac{7}{5} \left(1 + \frac{1}{10^2} + \frac{1 \cdot 3}{1 \cdot 2} \cdot \frac{1}{10^4} + \frac{1 \cdot 3 \cdot 5}{1 \cdot 2 \cdot 3} \cdot \frac{1}{10^6} + \dots \right)$$

22. If n is a positive integer and x is any non-zero real number, then prove that

$$C_0 + C_1 \cdot \frac{x}{2} + C_2 \cdot \frac{x^2}{3} + C_3 \cdot \frac{x^3}{4} + \dots + C_n \cdot \frac{x^n}{n+1} = \frac{(1+x)^{n+1} - 1}{(n+1)x}$$

23. Three urns have the following composition of balls :

Urn I : 1 white, 2 black

Urn II : 2 white, 1 black

Urn III : 2 white, 2 black

One of the urns is selected at random and a ball is drawn. It turns out to be white. Find the probability that it came from urn III.

24. The probability distribution of a random variable X is given below :

$$X = x_i \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$$

$$P(X = x_i) \quad k \quad 2k \quad 3k \quad 4k \quad 5k$$

Find the value of k and the mean and variance of X .