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I

Total No. of Questions : 24
Total No. of Printed Pages : 4

Regd. No.

Part-III

MATHEMATICS, Paper - II (B)

(English version)

Time : 3 Hours] ***[Max. Marks : 75***

Note : *This question paper consists of* ***three*** *sections* ***A, B*** *and* ***C.***

SECTION - A **10×2=20**

I . *Very short answer type questions.*

(i) Answer **all** questions.

(ii) Each question carries **TWO** marks.

1. Find the equation of the Circle, whose extremities of a diameter are $(-4, 3)$ and $(3, -4)$.

2. Find the centre and radius of the Sphere
$x^2 + y^2 + z^2 - 2x + 4y - 6z - 2 = 0$.

3. Find the value of K, if the lines $2x + 3y + 4 = 0$ and $x + y + k = 0$ are conjugate with respect to $y^2 = 8x$.

4. If the eccentricity of a hyperbola is $\frac{5}{4}$, then find the eccentricity of its conjugate hyperbola.

5. Find the n^{th} derivative of $f(x) = \log\ (4x^2 - 9)$ ..

6. Evaluate :

$$\int \frac{\sin(\tan^{-1} x)}{1+x^2}\, dx \ , \quad \text{for} \quad x \in \mathbb{R} \ .$$

7. Evaluate :

$$\int e^x\ (\sec x + \sec x\ \tan x)\, dx$$

on $I \subset R \setminus \left\{(2n+1)\,\pi/2 : n \in Z\right\}$

8. Evaluate :

$$\int_0^4 |\,2-x\,|\, dx.$$

9. Find the area bounded between the curves $y = x^2,\ y = 2x$.

10. Find the order and degree of the differential equation

$$\frac{d^2y}{dx^2} = \left[1+\left(\frac{dy}{dx}\right)^2\right]^{5/3}.$$

SECTION-B

5×4=20

II. *Short answer type questions.*

(i) Attempt **ANY FIVE** questions.

(ii) Each question carries **FOUR** marks.

11. If a point P is moving such that lengths of tangents drawn from P to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$ are in the ratio 2 : 3; then find the equation of the locus of P.

12. The normal at a point 't_1' on $y^2 = 4ax$ meets the parabola again at the point 't_2', then prove that $t_1 t_2 + t_1^2 + 2 = 0$.

13. Find the equations of the tangents to the hyperbola $x^2 - 4y^2 = 4$, which are *(i)* parallel, *(ii)* perpendicular to the line $x + 2y = 0$.

14. If PSQ is a chord passing through the focus S of a conic and 'l' is semi-latus rectum, show that $\frac{1}{SP} + \frac{1}{SQ} = \frac{2}{l}$.

15. Evaluate :

$$\int \frac{1}{5 + 4\cos 2x}\, dx .$$

16. Solve : $(xy^2 + x)dx + (yx^2 + y)dy = 0$.

17. Solve : $\left(1 + x^2\right) \frac{dy}{dx} + y = e^{\tan^{-1} x}$.

SECTION-C

5×7=35

III. *Long answer type questions.*

(i) Attempt **ANY FIVE** questions.

(ii) Each question carries **SEVEN** marks.

18. Find the equation of Circle passing through the points (3, 4), (3, 2), (1, 4).

19. Find the equation of the Circle which passes through the point $(0, -3)$ and intersects the circles given by the equations $x^2 + y^2 - 6x + 3y + 5 = 0$ and $x^2 + y^2 - x - 7y = 0$ orthogonally.

20. A chord PQ of an ellipse S = 0 subtends a right angle at the centre of the ellipse. Show that the point of intersection of tangents at P and Q lies on another ellipse $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{a^2}+\frac{1}{b^2}$.

21. If $y = e^{m \sin^{-1} x}$,

then prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+m^2)y_n=0$.

22. Obtain reduction formula for $I_n = \int \tan^n x \ dx$ for an integer $n \geq 2$,

and deduce the value of $\int \tan^6 x \, dx$.

23. Evaluate :

$$\int_0^1 \sqrt{\frac{1-x}{1+x}} \, dx.$$

24. Find the approximate value of π from $\int_0^1 \frac{1}{1+x^2} \, dx$

using Simpson's rule by dividing [0, 1] into 4 equal parts.
