MATHEMATICS PAPER IA - MARCH 2009.
ALGEBRA, VECTOR ALGEBRA AND TRIGONOMETRY

TIME: 3hrs **Max. Marks.75**

Note: This question paper consists of three sections A, B and C.

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS. **10X2 =20**

Note: Attempt all questions. Each question carries 2 marks.

1. If f and g are real valued functions defined by f(x) = 2x–1 and g(x) = x^2 then find i) (fg) (x) ii) (f+g+2) (x).
2. Find the domain and range of the function $f(x)=\frac{x}{2-3x}$
3. Let **a** = 2**i** + 4**j** – 5**k**, **b** = **i** + **j** + **k** and **c** = **j** + 2**k**. Find unit vector in the opposite direction of **a** + **b** + **c**.
4. OABC is a parallelogram. If **OA = a** and **OC = c** then find the vector equation of the side **BC**.
5. Find the radius of the sphere whose equation is $\bar{r}^2 = 2\bar{r}.\left(4\bar{i}-2\bar{j}+2\bar{k}\right)$.
6. If sinh x = 1/2, find the value of $\cosh 2x+\sinh 2x$.
7. If $\cos\theta+\sin\theta=\sqrt{2}\cos\theta$, then prove that $\cos\theta-\sin\theta=\sqrt{2}\sin\theta$.
8. Find the maximum and minimum values of $\cos\left(x+\frac{\pi}{3}\right)+2\sqrt{2}\sin\left(x+\frac{\pi}{3}\right)-3$.
9. If $\frac{a}{\cos A}=\frac{b}{\cos B}=\frac{c}{\cos C}$, then show that ΔABC is equilateral.
10. If $z_1=-1, z_2=i$, then find $\operatorname{Arg}\left(\frac{z_1}{z_2}\right)$.

SECTION B

SHORT ANSWER TYPE QUESTIONS **5X4 =20**

Note: Answer any FIVE questions. Each question carries 4 marks.

11. If **a, b, c** are linearly independent vectors, then show that **a** – 2**b** + 3**c**, –2**a** + 3**b** – 4**c**, – **b** + 2**c** are linearly dependent.
12. If **a** = 2**i** –+**j** - **k**, **b** = –**i** + 2**j** -4**k**, **c** = **i** + **j** +**k** , then find $\left(\bar{a}\times\bar{b}\right).\left(\bar{c}\times\bar{d}\right)$.
13. If A is not an integral multiple of , prove that cos A.cos 2A.cos 4A.cos 8A = $\frac{\sin 16A}{16\sin A}$.
14. Solve the equation $3Sin^{-1}\frac{2x}{1+x^2}-4\,Cos^{-1}\frac{1-x^2}{1+x^2}+2Tan^{-1}\frac{2x}{1-x^2}=\frac{\pi}{3}$
15. Prove that $\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\frac{s^2}{\Delta}$

16. Show that $\frac{\sin 6\theta}{\sin\theta} = 32\cos^5\theta - 32\cos^3\theta + 6\cos\theta$ when $\sin\theta \neq 0$.

17. Find the values of x in (-π, π) satisfying the equation $8^{1+\cos x+\cos^2 x+........\infty} = 4^3$

SECTION C

LONG ANSWER TYPE QUESTIONS **5X7 =35**

Note: Answer any Five of the following. Each question carries 7 marks.

18. Let $f : A \to B,\ g : B \to C$ be bijections. Then $gof : A \to C$ is a bijection.

19. Using mathematical induction prove that $1^2 + \left(1^2 + 2^2\right) + \left(1^2 + 2^2 + 3^2\right) + ...$ upto n terms $= \frac{n\left(n+1\right)^2\left(n+2\right)}{12}$

20. Find the shortest distance between the skew lines $\bar{r} = \left(6\bar{i} + 2\bar{j} + 2\bar{k}\right) + t\left(\bar{i} - 2\bar{j} + 2\bar{k}\right)$ and $\bar{r} = \left(-4\bar{i} - \bar{k}\right) + s\left(3\bar{i} - 2\bar{j} - 2\bar{k}\right)$

21. If A + B + C = 180^0, then prove that $\frac{\sin A + \sin B + \sin C}{\sin A + \sin B - \sin C} = \cot\frac{A}{2}\cot\frac{B}{2}$.

22. If $r_1 = 2$, $r_2 = 3$, $r_3 = 6$ and r = 1, prove that a = 3, b = 4 and c = 5.

23. From a point B on the level ground away from the foot of the hill AD, the top of the hill makes an angle of elevation a. From the point B, the point C is reached by moving a distance 'd' along a slant / slope which makes an angle g with the horizontal. If b is the angle of elevation of the top of the hill from C, find the height of the hill.

24. If n is an integer then show that $\left(1+i\right)^{2n} + \left(1-i\right)^{2n} = 2^{n+1}\cos\frac{n\pi}{2}$