266

III

Total No. of Questions : 24
Total No. of Printed Pages : 4

Regd. No.

Part-III

MATHEMATICS, Paper - II(A)

(English version)

Time : 3 Hours] ***[Max. Marks : 75***

Note : *This question paper contains* ***three*** *Sections* ***A, B*** *and* ***C.***

SECTION - A **10×2=20**

I . *Very short answer type questions.*

(i) Attempt **all** questions.

(ii) Each question carries **TWO** marks.

1. If α, β are the roots of the equation $ax^2 + bx + c = 0$, find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ in terms of a, b, c.

2. If the product of the roots of $4x^3 + 16x^2 - 9x - a = 0$ is 9, then find a.

3. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$ and $2X + A = B$, then find X.

4. If $A=\begin{bmatrix}2 & 0 & 1\\ -1 & 1 & 5\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 1 & 0\\ 0 & 1 & -2\end{bmatrix}$, then find $(AB')'$.

5. If ${}^{n}P_7 = 42 \cdot {}^{n}P_5$, find n.

6. If $10 \cdot {}^{n}C_2 = 3 \cdot {}^{n+1}C_3$, find n.

7. Find the set E of x for which the binomial expansion $(3-4x)^{3/4}$ is valid.

8. Find the co-efficient of x^2 in the series expansion of e^{3x+4}.

9. A page is opened at random from a book containing 200 pages. What is the probability that the number on the page is a perfect square ?

10. The probability that a person chosen at random is left-handed (in hand writing) is 0.1. What is the probability that in a group of 10 people, there is one who is left-handed ?

SECTION - B

5×4=20

II. *Short answer type questions.*

(i) Attempt **ANY FIVE** questions.

(ii) Each question carries **FOUR** marks.

11. Prove that $\frac{1}{3x+1}+\frac{1}{x+1}-\frac{1}{(3x+1)(x+1)}$ does not lie between 1 and 4, if x is real.

12. If $3A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ -2 & 2 & -1\end{bmatrix}$, then show that $A^{-1}=A^{T}$.

13. If the letters of the word 'RUBLE' are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the rank of the word 'LUBER'.

14. Prove that for $3 \le r \le n$,

$${}^{(n-3)}C_r + 3\cdot{}^{(n-3)}C_{r-1} + 3\cdot{}^{(n-3)}C_{r-2} + {}^{(n-3)}C_{r-3} = {}^{n}C_r$$

15. Resolve the fraction $\frac{2x^2+3x+4}{(x-1)(x^2+2)}$ into partial fractions.

16. Show that

$$\left(\frac{2x}{x^2+1}\right) + \frac{1}{3}\left(\frac{2x}{x^2+1}\right)^3 + \frac{1}{5}\left(\frac{2x}{x^2+1}\right)^5 + \ldots\ldots = \log_e\left(\frac{x+1}{x-1}\right).$$

17. The probability that Australia wins a match against India in a cricket game is given to be $\frac{1}{3}$. If India and Australia play 3 matches, what is the probability that *(i)* Australia will loose all the three matches ?
(ii) Australia will win atleast one match ?

SECTION - C

5×7=35

III. *Long answer type questions.*

(i) Attempt **ANY FIVE** questions.

(ii) Each question carries **SEVEN** marks.

18. Solve $x^4 + x^3 - 16x^2 - 4x + 48 = 0$, given that the product of two of the roots is 6.

19. Show that

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3.$$

20. Solve the following system of equations by Matrix inverse method.

$2x - y + 3z = 8$

$-x + 2y + z = 4$

$3x + y - 4z = 0$

21. If the coefficients of r^{th}, $(r+1)^{th}$ and $(r+2)^{nd}$ terms in the expansion of $(1+x)^n$ are in A.P., then show that $n^2 - (4r+1)n + 4r^2 - 2 = 0$.

22. If $x = \frac{5}{(2!)\cdot 3} + \frac{5\cdot 7}{(3!)\cdot 3^2} + \frac{5\cdot 7\cdot 9}{(4!)\cdot 3^3} + \ldots\ldots$,

then find the value of $x^2 + 4x$.

23. State and prove Addition theorem on Probability.

24. The random variable X has the following probability distribution.

$X = x$	0	1	2	3	4	5	6	7
$P(X = x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

find *(i)* k , *(ii)* the mean, and *(iii)* $P(0 < X < 5)$