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Total No. of Questions : 24
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Part-III

MATHEMATICS, Paper - II(A)

(English version)

Time : 3 Hours]

[Max. Marks : 75

Note : This question paper contains **three** Sections A, B and C.

SECTION - A

10×2=20

I. Very short answer type questions.

- (i) Answer **all** the questions.
(ii) Each question carries **two** marks.

- If the equation $x^2 - 15 - m(2x - 8) = 0$ has equal roots, find the value of 'm'.
- If $-1, 2, \alpha$ are the roots of the equation $2x^3 + x^2 - 7x - 6 = 0$, then find ' α '.

3. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x \end{bmatrix}$ and $\det A = 45$; then find 'x'.

4. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$; $B = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$ and $2X + A = B$, then find 'X'.

5. Find the number of ways of arranging the letters of the word "ENGINEERING".
6. If ${}^nC_5 = {}^nC_6$, then find ${}^{13}C_n$
7. Find the number of terms in the expansion of $(2x + 3y + z)^7$.
8. If $f(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots \infty$, then show that $f'(x) = \sinh x$.
9. If A, B are two events with $P(A \cup B) = 0.65$, $P(A \cap B) = 0.15$, then find the value of $P(A^c) + P(B^c)$.
10. If X is a Poisson variate with $P(X=0) = P(X=1) = k$, then show that $k = e^{-1}$.

SECTION-B**5×4=20****II. Short answer type questions.**

- (i) Answer **ANY FIVE** questions.
- (ii) Each question carries **Four** marks.

11. If x is real, prove that $\frac{x}{x^2 - 5x + 9}$ lies between $-\frac{1}{11}$ and 1.

12. If $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is a non-singular matrix, then show that A is

invertible and $A^{-1} = \frac{\text{Adj } A}{\det A}$.

13. Find the sum of all 4 digit numbers that can be formed using the digits 1, 3, 5, 7 and 9 (without repetition).

14. Find the number of ways of forming a Committee of 5 members, out of 6 Indians and 5 Americans, so that always the Indians will be in majority in the Committee.
15. Resolve $\frac{x^4}{(x-1)(x-2)}$ into partial fractions.
16. Show that
- $$1 + \frac{1}{3 \cdot 2^2} + \frac{1}{5 \cdot 2^4} + \frac{1}{7 \cdot 2^6} + \dots = \log_e 3.$$
17. A, B, C are three horses in a race. The probability of A to win the race is twice that of B and probability of B is twice that of C. What are the probabilities of A, B and C to win the race?

SECTION-C**5×7=35****III.** Long answer type questions.

- (i) Answer **ANY FIVE** questions.
- (ii) Each question carries **seven** marks.

18. Solve the equation

$$x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0.$$

19. Prove that

$$\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

20. Solve the equations

$$x + y + 4z = 6, \quad 3x + 2y - 2z = 9, \quad 5x + y + 2z = 13 \text{ by using Cramer's Rule.}$$

21. If n is a positive integer, prove that

$$\sum_{r=1}^n r^3 \left(\frac{{}^n C_r}{{}^n C_{r-1}} \right)^2 = \frac{n(n+1)^2(n+2)}{12}$$

22. If $x = \frac{5}{(2!)3} + \frac{5 \cdot 7}{(3!)3^2} + \frac{5 \cdot 7 \cdot 9}{(4!)3^3} + \dots$,

then find the value of $x^2 + 4x$.

23. State and prove Addition theorem on Probability.

24. The range of a random variable X is $\{0, 1, 2\}$.

Given that $P(X=0) = 3c^3$, $P(X=1) = 4c - 10c^2$, $P(X=2) = 5c - 1$,

(i) find the value of 'c', and

(ii) find $P(X < 1)$, $P(1 < X \leq 2)$, $P(0 < X \leq 3)$
