

Total No. of Questions - 24

Total No. of Printed Pages - 4

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## Part - III

## MATHEMATICS, Paper - II (B)

(Co-ordinate Geometry and Calculus)

(English Version)

Time : 3 hours

Max. Marks : 75

Note : This question paper consists of **three** sections A, B and C.

## SECTION A

## I. Very short answer type questions.

10 × 2 = 20

i) Answer **all** questions.ii) Each question carries **two** marks.

1. Find the other end of the diameter of the circle  $x^2 + y^2 - 8x - 8y + 27 = 0$  if one end of it is (2, 3).

2. Find the equation of the sphere whose center is (2, -3, 4) and radius is 5.

3. Find the equation of the parabola whose focus is S (1, -7) and vertex is A (1, -2).

4. Show that the angle between the two asymptotes of a hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  is  $2 \tan^{-1} \left( \frac{b}{a} \right)$  or  $2 \sec^{-1}(e)$ .

5. Find the  $n^{\text{th}}$  derivative of  $f(x) = \text{Log}(8x^3 + 36x^2 + 54x + 27)$  for all  $x > -\frac{3}{2}$ .

6. Evaluate  $\int \left(1 - \frac{1}{x^2}\right) e^{\left(x + \frac{1}{x}\right)} dx$ .
7. Evaluate :  $\int \frac{1}{(x+3)\sqrt{x+2}} dx$  on  $I \subset (-2, \infty)$ .
8. Evaluate :  $\int_0^{\pi/2} \frac{\sin^2 x - \cos^2 x}{\sin^3 x + \cos^3 x} dx$ .
9. Find the area bounded between the curves  $y^2 - 1 = 2x$  and  $x = 0$ .
10. Find the order and degree of the differential equation

$$\left[ \left( \frac{dy}{dx} \right)^{1/2} + \left( \frac{d^2y}{dx^2} \right)^{1/3} \right]^{1/4} = 0.$$

### SECTION B

#### II. Short answer type questions.

5 × 4 = 20

- i) Attempt **any five** questions.
- ii) Each question carries **four** marks.
11. Find the condition that the tangents drawn from the exterior point  $(g, f)$  to  $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$  are perpendicular to each other.
12. Find the equation of the parabola whose axis is parallel to Y-axis and which passes through the points  $(4, 5)$ ,  $(-2, 11)$  and  $(-4, 21)$ .
13. Find the eccentricity, foci and equation of the directrices of the hyperbola  $5x^2 - 4y^2 + 20x + 8y = 4$ .
14. If  $PP'$  and  $QQ'$  are two perpendicular focal chords of a conic, prove that  $\frac{1}{(SP)(SP')} + \frac{1}{(SQ)(SQ')}$  is constant.

15. Evaluate  $\int x \sqrt{1+x-x^2} dx$ .

16. Solve  $\frac{dy}{dx} - x \tan(y-x) = 1$ .

17. Solve  $(x+2y^3) \frac{dy}{dx} = y$ .

**SECTION C**

**III. Long answer type questions.**

**5 × 7 = 35**

- i) Attempt **any five** questions.
- ii) Each question carries **seven** marks.

18. Find the equation and center of the circle passing through the points (-2, 3), (2, -1) and (4, 0).

19. Find the equation of the circle which cuts the circles  $x^2 + y^2 + 2x + 4y + 1 = 0$ ,  $2x^2 + 2y^2 + 6x + 8y - 3 = 0$  and  $x^2 + y^2 - 2x + 6y - 3 = 0$  orthogonally.

20. The tangent and normal to the ellipse  $x^2 + 4y^2 = 4$  at a point  $P(\theta)$  on it meets the major axis in  $Q$  and  $R$  respectively. If  $0 < \theta < \frac{\pi}{2}$  and

$QR = 2$ , then show that  $\theta = \cos^{-1}\left(\frac{2}{3}\right)$ .

21. If  $y = \frac{\sinh^{-1} x}{\sqrt{1+x^2}}$  then show that  $(1+x^2)y_2 + 3xy_1 + y = 0$  and hence by using Leibnitz theorem, deduce that

$$(1+x^2)y_{n+2} + (2n+3)xy_{n+1} + (n+1)^2 y_n = 0.$$

22. Evaluate  $\int \frac{1}{\sin x + \sqrt{3} \cos x} dx$ .

23. Evaluate  $\int_0^1 \frac{\text{Log}(1+x)}{1+x^2} dx$ .

24. The velocity of a train which starts from rest is given by the following table, the time being recorded in minutes from the start and the speed in kilometers.

Minutes	2	4	6	8	10	12	14	16	18	20
Kmph	10	18	25	29	32	20	11	5	2	0

Estimate approximately the total distance run in 20 minutes by Simpson's rule and Trapezoidal rule.

SAKSHI