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Total No. of Questions – 24
Total No. of Printed Pages - 3

Regd. No.

Part - III
MATHEMATICS, Paper – II (B)
(English Version)

Time : 3 Hours] **[Max. Marks : 75**

Note : This question paper consists of **three** Sections **A**, **B** and **C**.

SECTION – A

$10 \times 2 = 20$

I. Very Short Answer Type questions.

(i) Answer **all** questions.

(ii) Each question carries **two** marks.

1. Find the equation of the circle passing through (3, 4) and having the centre at (–3, 4).

2. Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z - 2 = 0$.

3. Find the value of 'K' if points (1, 2), (K, –1) are conjugate with respect to the parabola $y^2 = 8x$.

4. If the eccentricity of a hyperbola is 5/4, then find the eccentricity of its conjugate hyperbola.

5. Find the n^{th} derivative of $f(x) = \sin 7x \cos x \; \forall \; x \in R$.

6. Evaluate $\int \left(x + \frac{1}{x}\right)^3 dx, x > 0$.

7. Evaluate $\int \frac{dx}{(x+1)(x+2)}$

8. Evaluate $\int_{2}^{3} \frac{2x}{1+x^2}\,dx$

9. Find the area of the region enclosed between $y = x^3 + 3$, $y = 0$, $x = -1$, $x = 2$.

10. Form the differential equation corresponding to $y = cx - 2c^2$, where 'c' is a parameter.

SECTION – B **$5 \times 4 = 20$**

II. Short Answer Type questions.

(i) Answer any **five** questions.

(ii) Each question carries **four** marks.

11. Find the angle between the tangents drawn from (3, 2) to the circle $x^2 + y^2 - 6x + 4y - 2 = 0$.

12. Find the condition for the line $y = mx + c$ to be a tangent to the parabola $x^2 = 4ay$.

13. Find the pole of the line $21x - 16y - 12 = 0$ with respect to the ellipse $3x^2 + 4y^2 = 12$.

14. If PSQ is a chord passing through the focus S of a conic and l is semi latus rectum, show that $\frac{1}{SP} + \frac{1}{SQ} = \frac{2}{l}$.

15. Evaluate $\int \frac{dx}{5 + 4\cos x}$

16. Solve the differential equation $(2x - y)dy = (2y - x)dx$

17. Solve the differential equation $\frac{dy}{dx} + y \tan x = \sin x$.

SECTION – C $5 \times 7 = 35$

III. Long Answer Type questions.

(i) Answer any **five** questions.

(ii) Each question carries **seven** marks.

18. Find the equation of a circle which passes through (4, 1), (6, 5) and having the centre on $4x + 3y - 24 = 0$.

19. Find the coordinates of the limiting points of the coaxial system to which the circles $x^2 + y^2 + 10x - 4y - 1 = 0$ and $x^2 + y^2 + 5x + y + 4 = 0$ are two members.

20. Show that the poles of the tangents to the circle $x^2 + y^2 = a^2 + b^2$ with respect to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ lies on $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{a^2 + b^2}$.

21. If $y = \cos(m \log x)$, $x > 0$, then show that $x^2y_2 + xy_1 + m^2y = 0$ and hence deduce that $x^2y_{n+2} + (2n+1)xy_{n+1} + (m^2 + n^2)y_n = 0$.

22. Obtain reduction formula for $I_n = \int \tan^n x \, dx$, n being a positive integer, $n \geq 2$ and deduce the value of $\int \tan^6 x \, dx$.

23. Show that $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx = \frac{\pi}{2\sqrt{2}} \log(\sqrt{2} + 1)$.

24. Calculate the approximate value of $\int_1^5 \frac{dx}{1+x}$, by taking n = 4 in the Simpson's rule.